FRACTIONAL ORDER POLE MODELS FOR ROBUST CONTROL ON INTERNET: www.PIDlab.com

Martin Čech[1] and Miloš Schlegel[1]
[1]Department of Cybernetics,
University of West Bohemia in Pilsen
Pilsen, Czech Republic,
mcech@kky.zcu.cz,
schlegel@kky.zcu.cz

Abstract: This paper describes the simple Internet tool for fractional order pole models - the Java applet free accessible on www.PIDlab.com. The applet implements procedures described in the authors' previous works, where the value set approach to fractional order pole systems was presented for the case, when following characteristic numbers of the process were obtained: a) one sample of the process frequency response, b) first three moments of the process impulse response. Using the applet, we can easily compute value sets and find the upper hard bound of the value sets for the case, when one/two frequency response samples and first three moments of the impulse response are known simultaneously. Then we can determine processes on the value set boundary which will be further important for robust controller design.

Key words: fractional-order pole model, identification, model set, robust controller design, Java applet

1 Introduction

It is well known from the classical control theory [1] that process controllers can be designed on the base of a few points of the process frequency response or on the base of some rectangle pulse response. In this direction, the limit method is the popular Ziegler-Nichols method which uses only one so called ultimate point or two numbers obtained from step response. However, these traditional methods are neither systematic nor guarantee fulfillment of design specifications for an exactly given class of process transfer functions. It was shown in previous works [5-6], that after adding the key *a priori* information about the process transfer function to experimental data one obtains closed areas – value sets - in the frequency domain representing generalized points of process frequency response for each frequency. Using these value sets, one can further design the robust controller guaranteeing fulfillment of some design specifications (e.g. gain and phase margins) for all processes satisfying *a priori* restriction. In accordance with the majority of works in the process control field, it was assumed that the real process can be described by a multiple fractional order pole model in the form

$$F(s) = \frac{K}{\prod_{i=1}^{p} (t_i s + 1)^{n_i} s^{n_p}}, \qquad t_i > 0, i = 1, \mathbf{K}, p-1, \tag{1}$$

where K >0, p is an arbitrary integer and $n_i \geq 0, \quad i = 1, 2, \mathbf{K}, p$ are real numbers. It is believed, that the class of transfer functions (1) is sufficiently rich at least for control purposes because it includes all integer order lag/dead time processes of arbitrary order. The main result of previous works was the explicit parameterization of the value set boundary for the model family consistent with the *a priori* information (1) and a) one experimentally obtained point of frequency response b) first three moments of the process impulse response. The presented results have important applications in design of robust controllers and automatic tuning procedures [2]. Therefore, we decided to create a Java applet accessible on Internet, which allows computing corresponding value sets for experimental data a) and b) for arbitrary frequency.

The paper is organized as follows. In Section 2, the previous results are remembered. Section 3 gives a brief user description of the Java applet. Practical example is shown in Section 4. Section 5 contains some concluding remarks and ideas for further work.

2 Previous results

The previous works deal with two special cases of the general frequency response interpolation problem, where the interpolation conditions Π on the transfer function *F(s)* are in the form

$$\Pi = \{F^{(l)}(s)|_{s=jw_i} = P_{il}, \quad i = 0, 1, \mathbf{K}, k, \quad l = 0, 1, \mathbf{K}, m_i - 1\}, \tag{2}$$

where k and m_i are given integers and $P_{il} \in C$.

In other words, the frequency samples at frequencies $\omega_1 \ldots \omega_k$ are given and also some derivatives of the frequency response at these points can be known.

Definition 1 (Process Model Set). *The transfer function F(s) is admissible, if following conditions are satisfied*

(i) Transfer function F(s) is in the form (1), $n_i \geq m, \; i = 1, 2, \mathbf{K}, p, \quad \sum_{i=1}^{p} n_i \leq n$ *, where n is the total order of the process and m is the minimum order of each pole*

(ii) Transfer function F(s) satisfies interpolation conditions (2)

The set of all admissible systems will be called process model set and denoted by $S_p^{n,m}(\Pi)$

Definition 2 (Value set). *The set* $V_p^{n,m}(w) = \{F(jw) : F(s) \in S_p^{n,m}(\Pi)\}$ *in the complex plane will be called the value set of the model set* $S_p^{n,m}(\Pi)$ *at frequency* ω>0.

1.1 First three moments of the impulse response

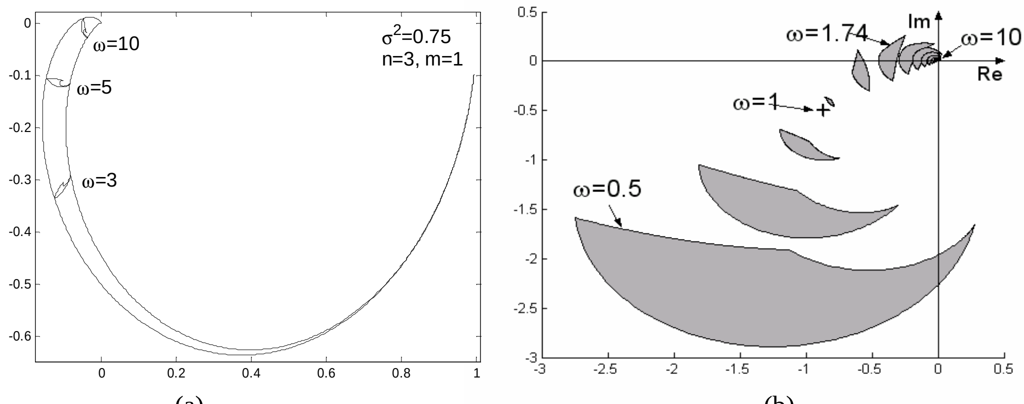
In [5], the interpolation condition is the point at frequency ω = 0 and first three derivatives at this frequency. As it was shown, these three numbers can be substituted by first three moments of the impulse response

$$m_i = \int_0^\infty t^i h(t)dt, \qquad i = 0,1,2\,, \tag{3}$$

which can be also equivalently substituted by another three numbers κ, μ, σ^2 defined as

$$k = m_0 = F'(0), \qquad m = \frac{m_1}{m_0} = -\frac{F'(0)}{F(0)}, \qquad s^2 = \frac{m_2}{m_0} - \frac{m_1^2}{m_0^2} = \frac{F''(0)}{F(0)} - \frac{F'(0)^2}{F(0)^2}\,. \tag{4}$$

The band of all admissible transfer functions frequency responses and value sets for some frequencies are shown if Fig. 1a).



(a) (b)

Fig. 1. Examples of the value sets for special types of interpolating conditions

1.2 One sample of the process frequency response

In [6], it is assumed, that one sample of the process frequency response was obtained and no derivatives of the frequency response at this point are known. The example of value sets for some frequencies is shown in Fig. 1b.

3 Java applet „*Fractional PID laboratory*"

The applet imports the basic axes functionality from the "PID Controller Designer" and is divided into four basic windows.

PC – Process and controller parameters (upper left corner). In the "Model set" tab, the user can set the measured characteristic numbers of the process in the form of the numbers (4) or in the form or frequency response sample (amplitude, phase, frequency). The order restriction *n*, *m* can be changed in the first panel. After choosing the type of the process (fractional and/or integer order), the value sets for frequencies defined in the text field will be painted immediately in the **FD** window. If any characteristic number or order restriction is changed, the value sets for all frequencies are recomputed and repainted. If one clicks on any value set, the process generating the corresponding value set point is added into the process list in the "Process" tab and can be further used for robust controller design. The process title, color and visibility can be edited in the "Process" tab and processes can be deleted there.

FD – Frequency domain (lower left corner). In this window, the value sets and the process frequency responses are painted. The frequency response sample amplitude and phase can be changed easily by the sample cross mouse dragging.

RR – Robustness regions (upper right corner).

LP – Loop performance (lower right corner).

The last two windows are useful for robust fractional PID controller [3,4] design. This topic is omitted because of the restricted range of this paper.

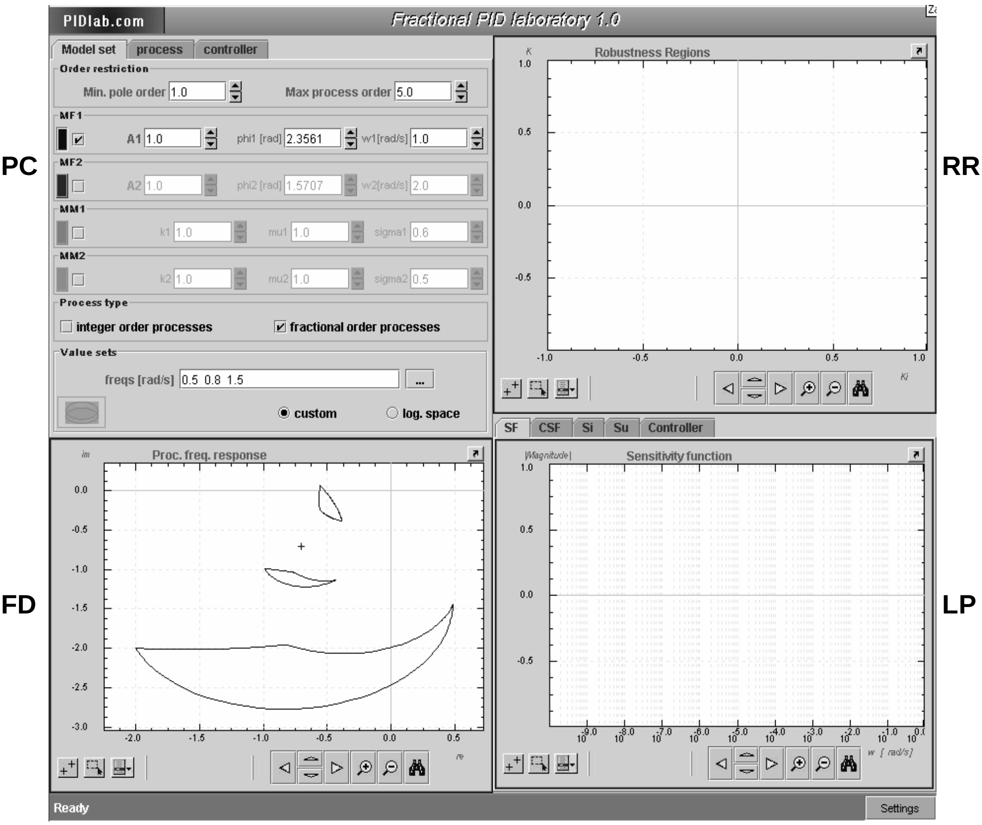
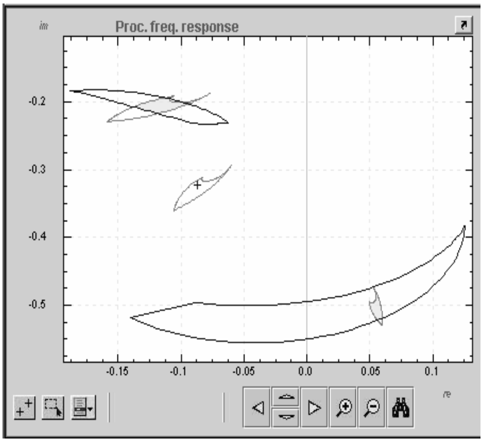
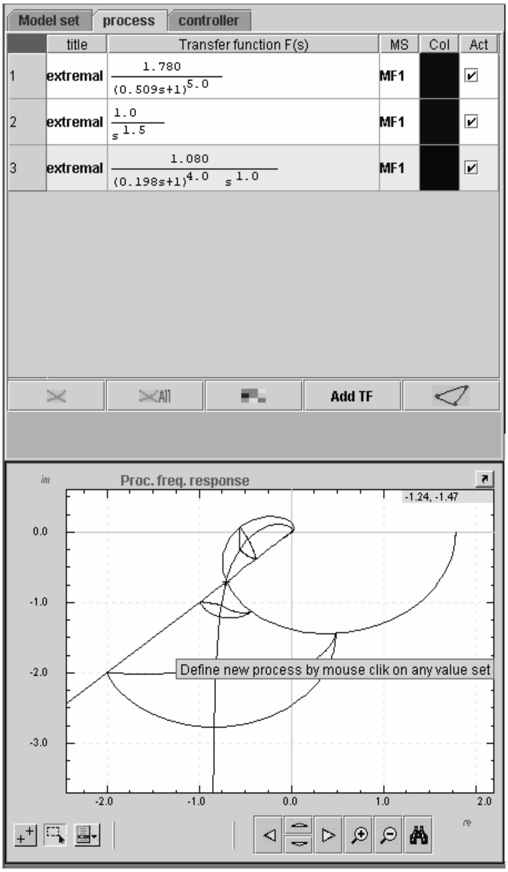


Fig. 2. General view of the Java applet

In “Process” tab, the extremal processes Important for controller design, can be obtained by “Vertex processes button”. In process list, it can be checked, that the extremal processes are created by one fractional order pole and fractional order integrator (Fig. 3.b).





a) b)

Fig. 3. a) intersection of value sets b) extremal processes important for robust controller design

4 Practical example

Consider, that the following characteristic numbers were obtained in the identification experiment

$$\{k = 1, \quad m = 1, \quad s^2 = 0.6\}; \quad \{p_1 = 1.83e^{-j0.33}, w = 3\} \tag{4}$$

Using the applet, the upper hard bound of the value set boundary can be found as an intersection of two value sets (Fig. 3a). It can be seen, that combining several experimental data leads to the significant model set reduction in the frequency domain

5 Conclusions

The main aim of this paper is to present the simple internet tool – Java applet free accessible on www.PIDlab.com – for fractional order pole models. The applet implements the methods presented in previous works, where it was assumed, that characteristic numbers of the process are known in the form of one sample of process frequency response of first three moments of the impulse response. The characteristic numbers are combined with *a priori* information about the transfer function (fractional-order pole model with/without restriction on the total model order and the minimum pole order). Then the value sets can be painted in frequency domain for each frequency. These characteristic numbers can be easily obtained on the real process, therefore we believe, that the applet can be useful for industrial practitioners. It is also shown, that combining one/two samples of frequency response together with moment experimental data leads to the significant reduction of the model uncertainty. However, then only hard bounds of the value set boundary can be computed as an intersection of corresponding value sets. We hope that in the future the fractional PID controller design will be completed by adding time simulations to the applet.

This work was particularly supported by MŠMT ČR – project no. MSM 235200004 and GAČR – project no. 102/02/0425.

References

[1] Aström K.J., Hägglund T.: *PID Controllers: Theory Design and Tuning.* NC: Instrument Society of America, Research Triangle Park, 1995.

[2] Schlegel M., Balda P., Štětina M.: *Robust PID autotuner- method of moments.* Automatizace, Vol. 46, No.4.

[3] Podlubny I.: *Fractional-order systems and* $PI^{\lambda}D^{\mu}$ *controllers.* IEEE Trans. Automatic control, vol. 44, no. 1, pp. 208-214, 1999.

[4] Schlegel M., Mertl J., Čech M.: *Generalized robustness regions for PID controller.* Process Control 03 Summaries Volume, High Tatras, Slovak Republic, June 2003, s. 108, ISBN 80-227-1902-1.

[5] Schlegel M., Čech M.: *Fractal system identification for robust control: method of moments.*ICCC 2004 summaries volume, May 2004, Zakopane, Poland.

[6] Schlegel M., Čech M.: *Computing value sets from one of the frequency response with applications.* IFAC 2005, 4-8 July 2005, Praque, Czech Republic.