Design of PID Controllers: H_∞ Region Approach

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An introduction to PID Hinf Designer

www.pidlab.com/pidhinf

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Contents

PID design method

- For a long time, the development of PID controller design methods has been the goal of the control community. Despite that manual model-free tuning of controllers is still the most commonly used PID design method in industry.
- Tuning rules (Ziegler-Nichols, Lambda tuning, AMIGO method [1], Internal model control, Skogestad's SIMC method [2], …)

  Universal relations between model and controller parameters.

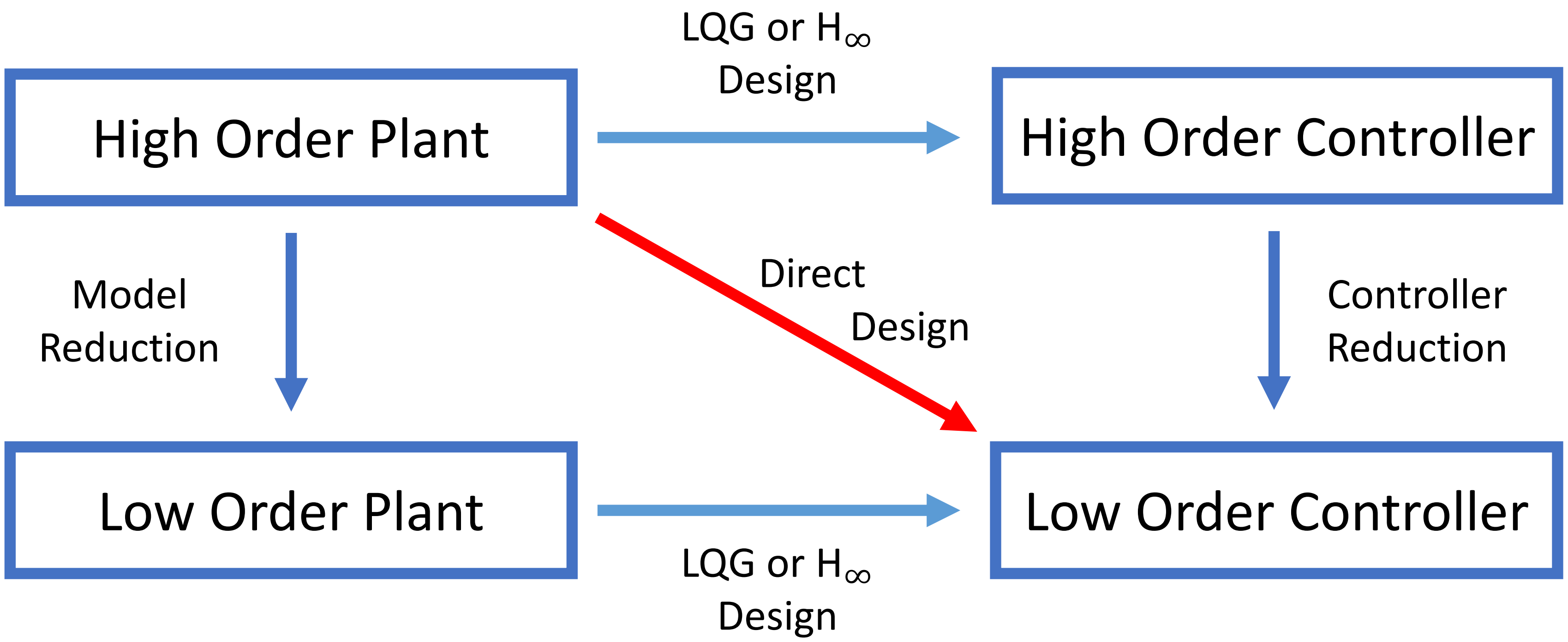
- Optimization-based method (MIGO [3], SWORD [4], MATLAB pidTuner, hinfstruct)

  Treats each process model individually.

[1] Astrom, K.J. and T. Hagglund: Advanced PID Control. ISA, 2006, ISBN 1-55617-942-1
[2] Sigurd Skogestad and Chriss Grimholt. "The SIMC Method for Smooth PIDController Tuning". PIDControl in the Third Millennium.Springer. 2012
[3] Astrom, K.J., Panagopoulos, H., Hagglund, T.: Design of PI Controllers based on Non-Convex Optimalization. Automatica, Vol. 34, No. 5, pp. 585-601, 1998.
[4] Garpinger O. Analysis and Design of Software-Based Optimal PID Controllers. PhD Thesis, Department of Automatic Control Lund University, 2015.

There exists no generally accepted design method for PID controller

The design procedures associated with modern control theory (Hinf, LQR) provide high order controllers. Practice prefers simple controllers.

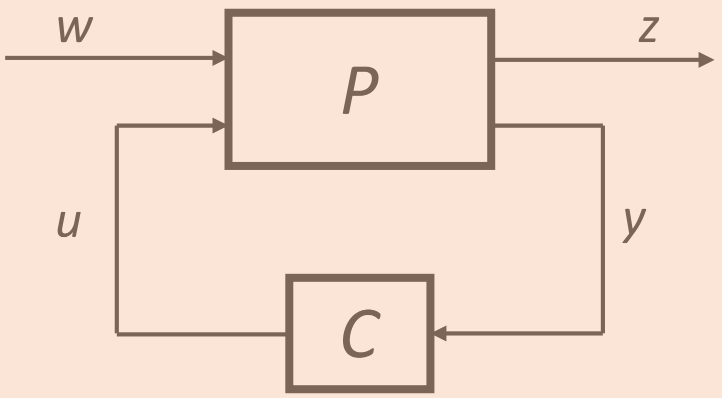


Anderson, B.D.O.: Controller Design Moving from Theory to Practice. 1992 Bode Prize Lecture.

Requirements for effective design method

- It should be applicable to a wide range of systems (i.e. stable/unstable/non minimal phase/oscillatory process transfer functions)
- It should have the possibility to introduce specifications that capture the essence of real control problems (i.e. robustness/performance trade-off, servo/regulator problem)
- The method should be robust in the sense that it provides controller parameters if they exist, or if the specifications cannot be meet an appropriate diagnosis should be presented

The general H∞ Control Problem



$$\text{minimize} \quad \left\| H_{w\to z}(P,C) \right\|_\infty$$
$$\text{subject to} \quad C \text{ stabilizes P internally}$$
$$C \in \mathbf{C}$$

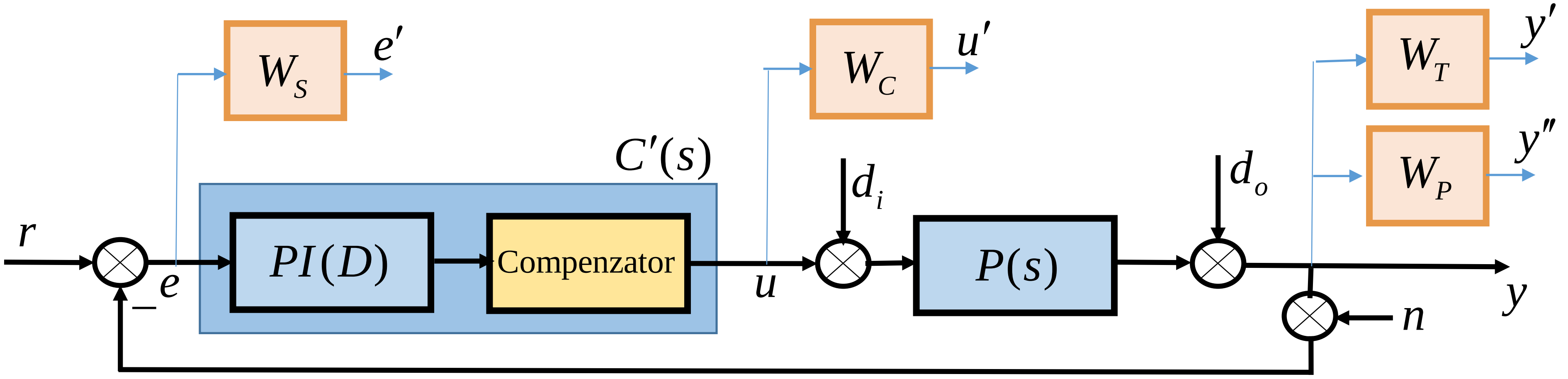
$P = P(s)$	Given a real rational transfer matrix called the plant
$C \in \mathbf{C}$	Searched controller from the controller space $\mathbf{C}$
$H_{w\to z}(P,C)$	The closed-loop performance or robustness transfer matrix

In our considered case, $H \equiv H_{w\to z}(P,C)$ is a scalar function and it holds

$$\left\| H \right\|_\infty \triangleq \sup_{w\neq 0} \frac{\left\| Hw \right\|_2}{\left\| w \right\|_2} = \sup_{w\neq 0} \frac{\left\| z \right\|_2}{\left\| w \right\|_2} = \max_\omega \bar{\sigma}\left(H(j\omega)\right)$$

$$\left\| z \right\|_2 \triangleq \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} Tr\left(H(j\omega)H^H(j\omega)d\omega\right) \right)^{\frac{1}{2}}$$

The H∞ Control Problem considered



Find all controllers C' for which it holds

$$\left\| H_{w \to z}(P, C') \right\|_\infty \leq \gamma$$

subject to C' stabilizes P internally

$$C' \triangleq C_{PID} \cdot C_{comp} \in \mathbf{C}.$$

The performance or robustness channel $H \equiv H_{w \to z}(P, C)$ is a scalar weiting closed-loop sensitivity function and it holds

$$\|H\|_\infty \triangleq \sup_{w \neq 0} \frac{\|Hw\|_2}{\|w\|_2} = \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2} = \max_\omega |H(j\omega)|$$

$$\|z\|_2 \triangleq \left(\int_{-\infty}^{+\infty} z^2(t)\, dt \right)^{\frac{1}{2}} = \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} |Z(j\omega)|^2 d\omega \right)^{\frac{1}{2}}$$

PID Hinf Designer

(www.pidlab.com/pidhinf)

- PID Hinf Designer is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE, ISE and Hinf robustness constraints.
- PID Hinf Designer can be used for a wide range of proces models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, …) and also for so-called model sets created from any number of process transfer functions.
- Supported design specifications reflect the essence of real control problems. Optimization of integral criteria IE, ISE, IAE, ITAE under Hinf constraints is supported for both load disturbance attenuation and set-point tracking problems).
- Designing of PI(D) controller with typical specifications using PID Hinf Designer is a routine procedure that does not require deeper knowledge of control theory from the user.
- With more skills and efforts from the designer it should be possible using PID Hinf Designer to design high performance PID controllers extended with a suitable linear compensator (Cascade Controller, Resonant Controller, Smith Predictor, Repetitive Control, …).
- PID Hinf Designer also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one point of the frequency response). Moreover, the non-standard moment model set provided by the PIDMA-autotuner from the company REX Controls is also supported.

PID Hinf Designer Options

(www.pidlab.com/pidhinf)

PID Control

- Load Disturbance Attenuation
- Set-point Tracking Problems

Advanced PID Control

Cascade Control, Resonant Control, Repetitive Control, PID Control for Complex Systems, ...

PID Controller

1DOF PI Controller

2DOF PI Controller

1DOF PID Controller

2DOF PID Controller

Arbitrary Rational Transfer Function (RTF) without poles on the imaginary axis

- second order compensator
- lead-lag compensator

Static or Integrating Process (Arbitrary strictly proper Rational Transfer Function (RTF) without poles on the imaginary axis except one for integrating processes)

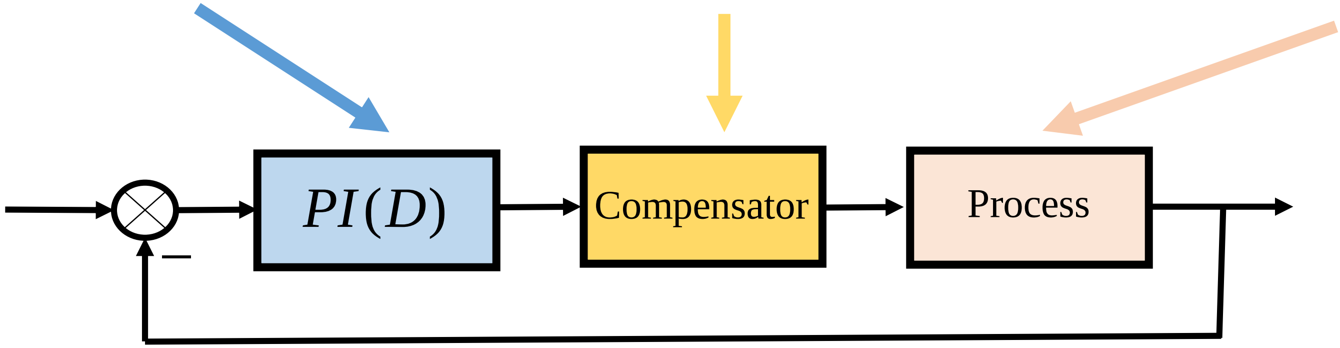
RTF + Time Delay (RTFTD)

Assembled Model

Experimental Input/Output Data

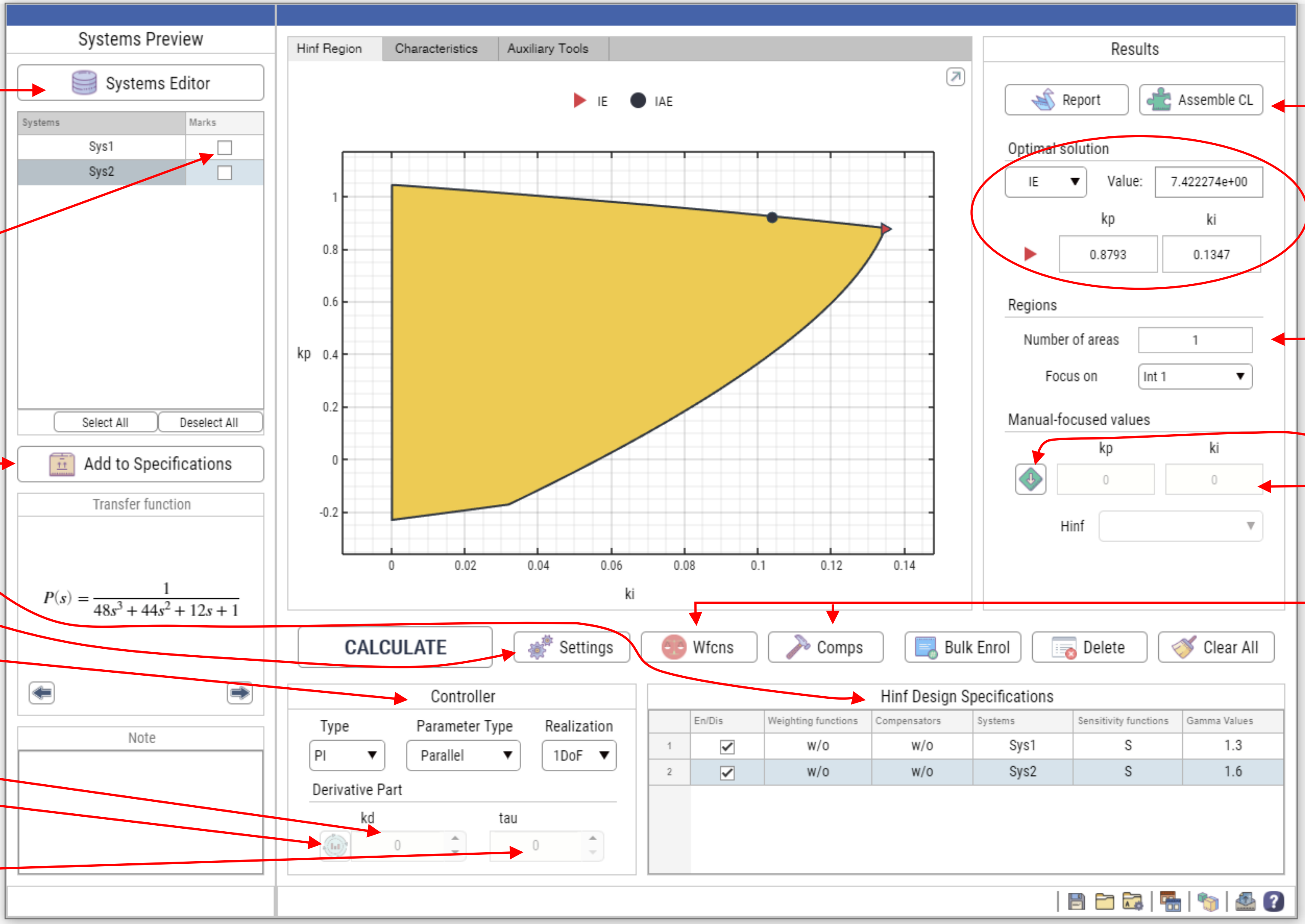
Model Set (Set of RTFTD)

- One Frequency Point Process Model
- (κ, μ, σ) Process Model
- Model with Parametric Uncertainty



PID Hinf Designer GUI

(www.pidlab.com/pidhinf)



Entering transfer functions of processes

Selection of a model set

Enter H∞ limitations

1

2

Selection of the design criterion

Select the controller type

Estimate kd (PID) :
Manually
Automatically

Estimate tau (PID) :
Manually

Create a closed-loop assembled transfer function (e.g. for cascade control)

The resulting controller

H∞ region selection

1

2

Manual tuning of the controller

Enter weighting functions and compensators

Select weighting functions, compensators, systems, sensitivity functions and values of H∞ limitations

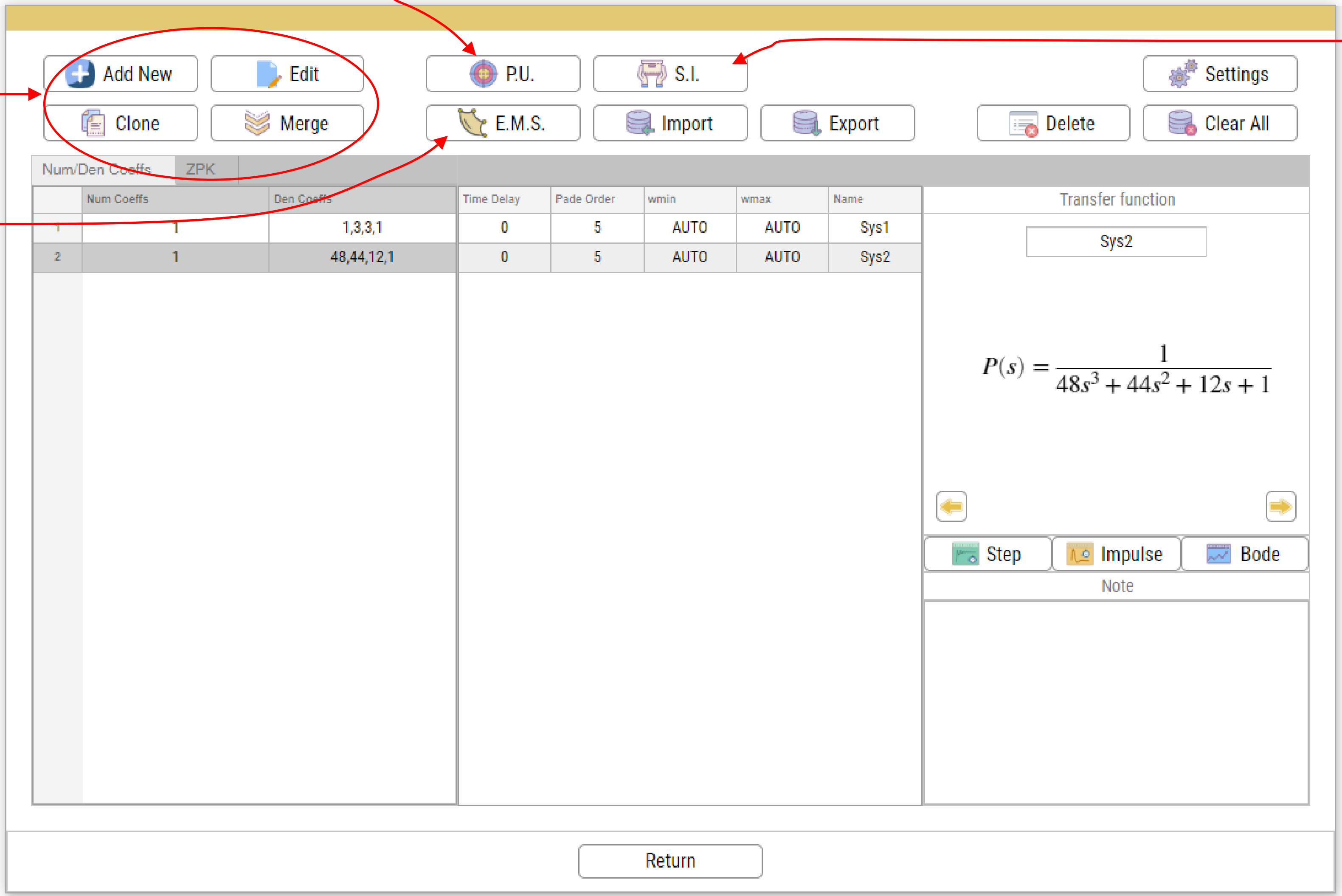
PID Hinf Designer GUI – Systems Editor

Parameter Uncertainty Model Set (See Appendix D)

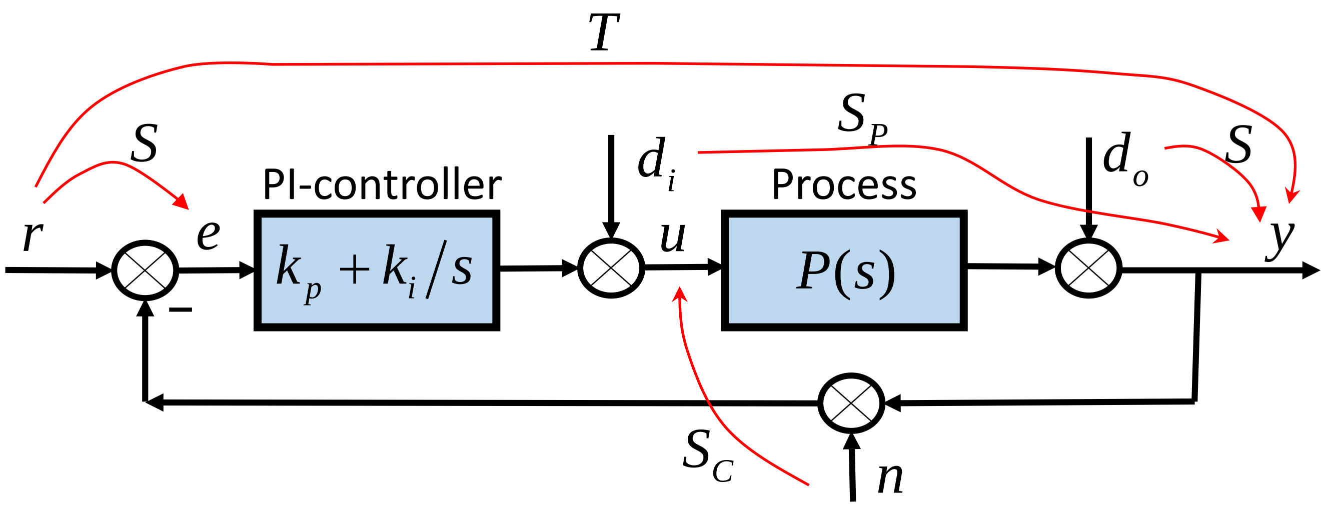
Rational Transfer Function + Time Delay

Experimentally Determined Model Set (See Appendix E)

System Identification Experimental I/O data (See Appendix F)



Parameter Plane Formulation of Basic PI-Controller Design Problem



$H(s, k = [k_p, k_i]) \triangleq W(s)S_*(s,k),\ S_* \in \{S, T, S_C, S_P\}$ .... weighting sensitivity function

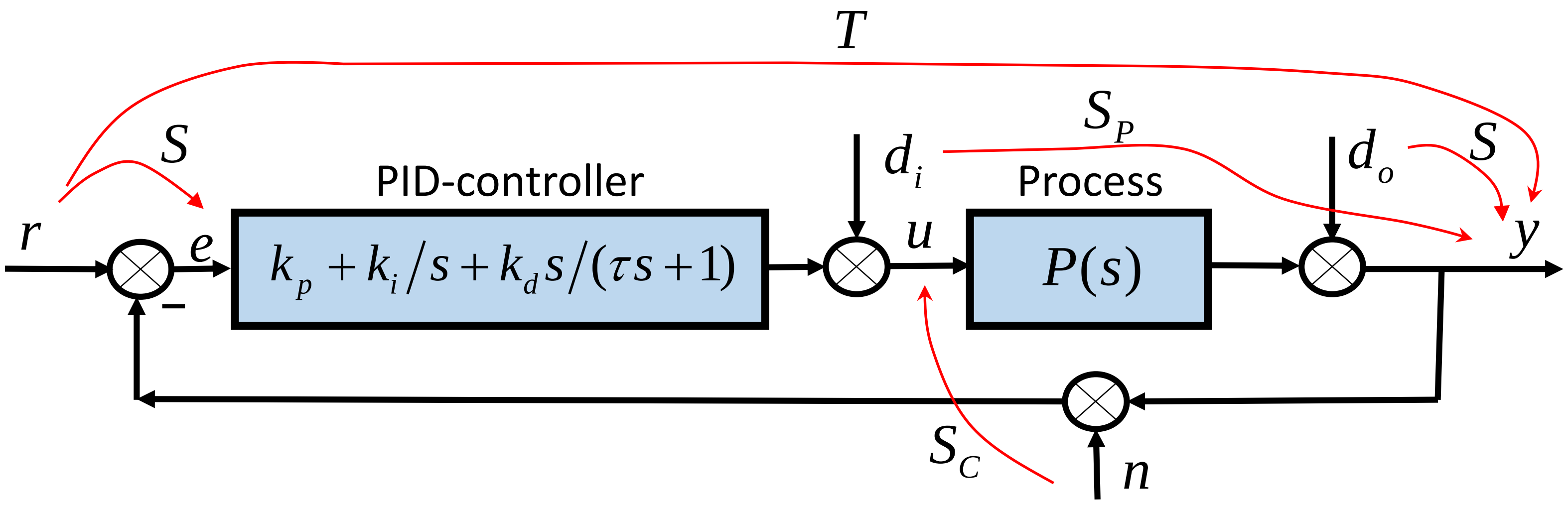
$\mathrm{K} \triangleq \left\{ \left[k_p, k_i\right] :\ \|H(s,k)\|_\infty \triangleq \sup_\omega |H(j\omega, k)| \le \gamma, \text{ the closed-loop is stable} \right\}$ ...... H_∞-region in the parameter plane

1) Find the H_∞- region K in the k_i-k_p plane. (See Appendix A for details.)

2) Find the optimal PI-controller in the H_∞- region K with respect to the criterion $IAE \triangleq \int_0^\infty |e(t)|\, dt$

for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

Parameter Plane Formulation of Basic PID-Controller Design Problem



$H(s,k=[k_p,k_i,k_d,\tau]) \triangleq W(s)S_*(s,k),\ S_* \in \{S,T,S_C,S_P\}$ .... weighting sensitivity function

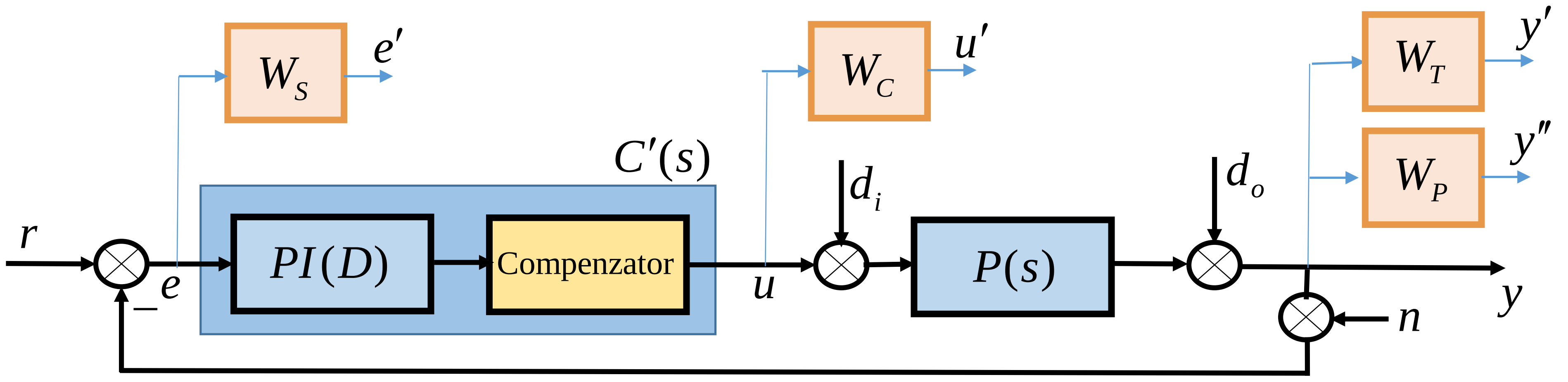
$\mathrm{K}_{[k_d,\tau]} \triangleq \left\{ \left[k_p,k_i\right]:\ \|H(s,k)\|_\infty \triangleq \sup_\omega |H(j\omega,k)| \le \gamma,\ \text{the closed-loop is stable} \right\}$ ... H_∞-region in the parameter plane k_i-k_p for the fixed k_d and τ

1) Choose the derivative gain k_d and the time constant τ manually or with the help of a built-in function. (See Appendix B for details.)

2) Find the H_∞- region $\mathrm{K}_{[k_d,\tau]}$ in the k_i-k_p plane.

3) Find the optimal PID-controller in the H_∞- region $\mathrm{K}_{[k_d,\tau]}$ with respect to the criterion $IAE \triangleq \int_0^\infty |e(t)|dt$ for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

H_∞ limitations supported



$$\|H(s)\|_\infty \le \gamma \Leftrightarrow |H(j\omega)| \le \gamma, \forall \omega \in [0, \infty)$$

Sensitivity functions (gang of four)

$$S = \frac{1}{1 + C'P}, T = C'PS, S_C = C'S, S_P = PS$$

Weighting functions

$$W_S(s), W_T(s), W_C(s), W_P(s),$$

Servo problem

(set-point tracking)

$r \to e' : \|W_S S\|_\infty \le M_S$

$r \to y' : \|W_T T\|_\infty \le M_T$

$n \to u' : \|W_C S_C\|_\infty \le M_C$

Regulator problem

(load disturbance rejection)

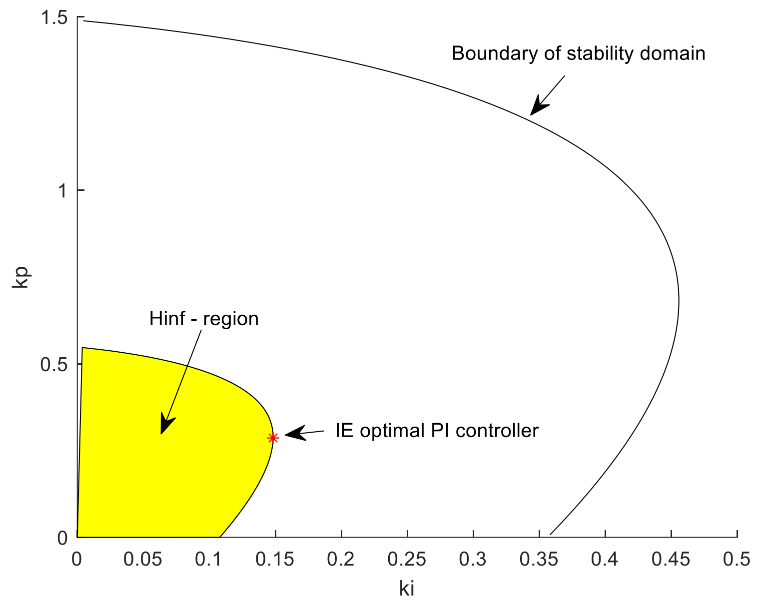
$d_i \to y'' : \|W_P S_P\|_\infty \le M_P$

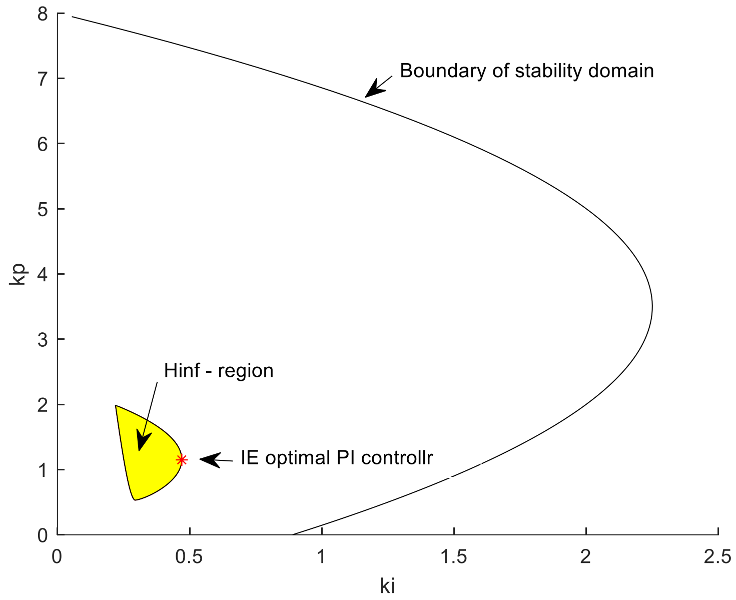
$d_o \to e' : \|W_S S\|_\infty \le M_T$

$n \to u' : \|W_C S_C\|_\infty \le M_C$

H_∞-Region in the Parametric Plane $k_i - k_p$

(It contains all PI controllers that meet the specified H∞ limitations)





Finding the H_∞ - region

$$K \triangleq \left\{ k = \left[k_p, k_i \right] : \left\| H(s,k) \right\|_\infty \leq \gamma, \text{ the closed-loop is stable} \right\}$$

is generally a very difficult problem. PID Hinf Designer (www.pidlab.com/pidhinf) is the first software tool available to fully address this issue.

Example of Simple Design specification of PI-controller for FOPDT system

Proces transfer function: $P(s) = \dfrac{e^{-s}}{s+1}$

Controller transfer function: $C_{PI}(s) = K\left(1 + \dfrac{1}{T_i s}\right) = k_p + \dfrac{k_i}{s}$

Sensitivity function: $S(s) = \dfrac{1}{1 + C_{PI}(s)P(s)}$

Weighting function: $W_S(s) = 1$

Type of control problem: regulator problem (load step disturbance rejection)

Design specification: $IAE = \min_{C_{PI}} \int_0^\infty |e(t)|\,dt$

subject to $\|S(s)\|_\infty \leq M_S \iff |S(j\omega)| \leq M_S,\ \forall \omega \in [0, \infty)$

PID Hinf Designer

Input:

$$P(s) = \frac{e^{-s}}{s+1}$$

PI-controller $\left(C_{PI}(s) = k_p + \frac{k_i}{s} \right)$

$M_S = 1.6$

Output:

$IE: \quad k_p = 0.463, \quad k_i = 0.509$

$IAE: \quad k_p = 0.565, \quad k_i = 0.488$

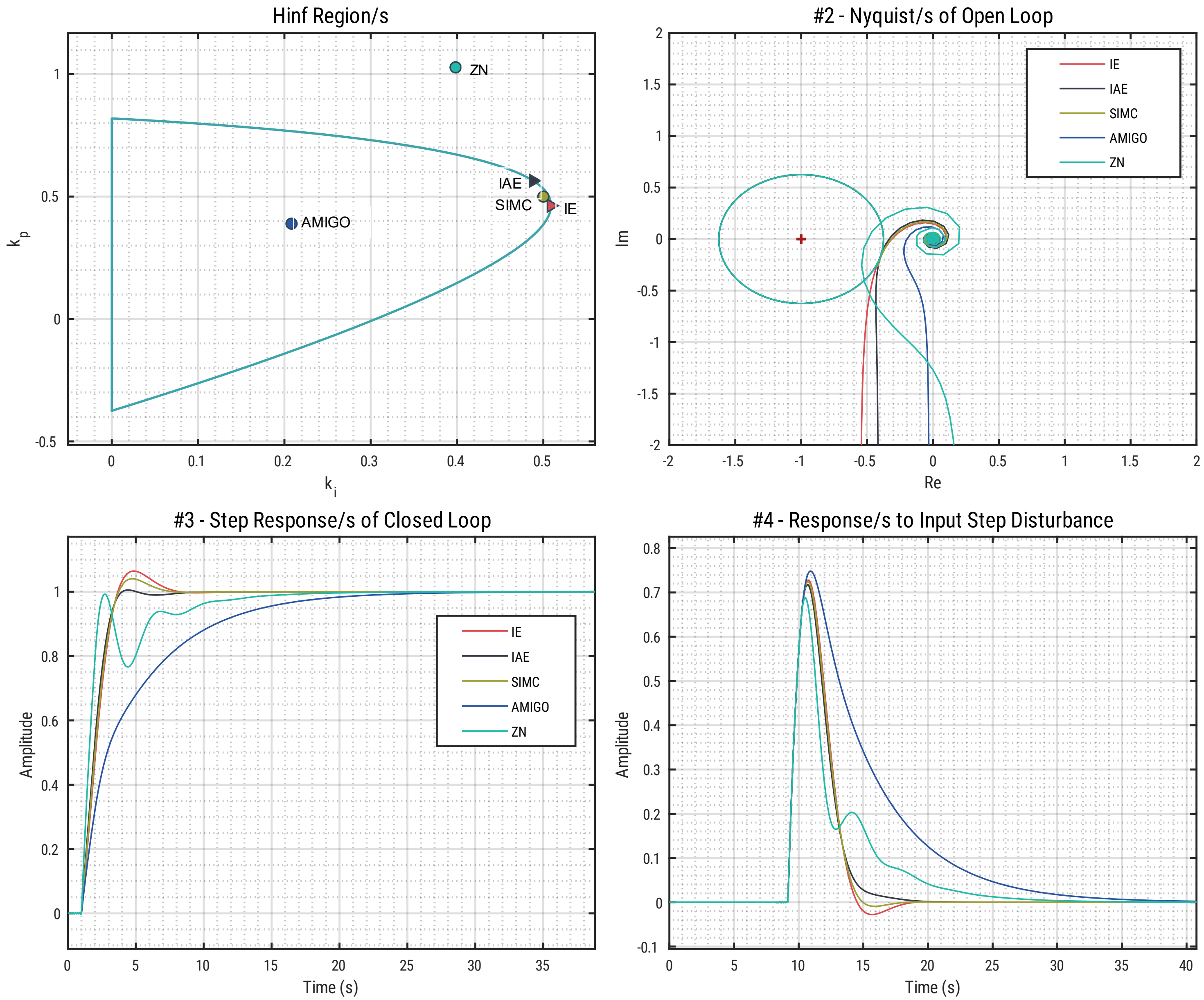
$ITAE: \quad k_p = 0.557, \quad k_i = 0.492$

Optional output:

ZN Ziegler-Nicols (1942, step response)

SIMC Skogestad (2012)

AMIGO Hagglund and Astrom (2004)



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More General Formulation of Design Problem

(fully supported by PID Hinf Designer, www.pidlab.com/pidhinf)

$\mathbf{P} = \{P_1, P_2, \ldots, P_k\}$ model set of transfer functions

$P \in \mathbf{P}$ process transfer function

$C \in \{C_{PI}, C_{PID}\}$ controller transfer function

$S = \frac{1}{1+CP},\ T = \frac{CP}{1+CP},\ S_C = \frac{CP}{1+CP},\ S_P = \frac{P}{1+CP}$ loop sensitivity transfer functions

$\mathbf{I} = \{IE, IAE, ITAE, ISE\}$ design criterion set

$$IE \triangleq \int_0^\infty e(t)dt,\ IAE \triangleq \int_0^\infty |e(t)|dt,\ ITAE \triangleq \int_0^\infty t|e(t)|dt,\ ISE \triangleq \int_0^\infty e^2(t)dt$$

$I \in \mathbf{I}$ design criterion selected

W_S, W_T, W_C, W_P weighting functions

Controller Robust Design Problem

$$\min_C \max_{P \in \mathbf{P}} I$$

subject to the H_∞ limitations

$$\forall P \in \mathbf{P}:\ \|W_S S\|_\infty \le M_S,\ \|W_T T\|_\infty \le M_S,\ \|W_C S_C\|_\infty \le M_C,\ \|W_P S_P\|_\infty \le M_P.$$

Example of Design Specification of Robust PI-controller for Process Model Set

Proces model set: $\mathbf{P} \triangleq \left\{ P_1(s) = \dfrac{-0.0216s + 0.0031}{s^2 + 0.457s + 0.0868} e^{-0.166s},\ P_2(s) = \dfrac{-0.0174s + 0.0046}{s^2 + 0.5978s + 0.0445} e^{-0.166s} \right\}$

Controller transfer function: $C_{PI}(s) = K\left(1 + \dfrac{1}{T_i s}\right) = k_p + \dfrac{k_i}{s}$

Sensitivity functions: $S_i(s) = \dfrac{1}{1 + C_{PI}(s)P_i(s)},\ i = 1,2$

Weighting functions: $W_i(s) = 1,\ i = 1,2$

Type of control problem: regulator problem (load step disturbance rejection)

Design specification: $\min_{C_{PI}} \max_{i \in \{1,2\}} \int_0^\infty |e_i(t)|\, dt$

subject to $\|S_i(s)\|_\infty \le M_S,\ i = 1,\ldots,2 \ \Leftrightarrow\ |S_i(j\omega)| \le M_S,\ i = 1,2,\ \forall \omega \in [0,\infty)$

PID Hinf Designer

Input:

$$P_1(s) = \frac{-0.0216s + 0.031}{s^2 + 0.457s + 0.0868} e^{-0.166s}$$

$$P_2(s) = \frac{-0.0174s + 0.0445}{s^2 + 0.5978s + 0.0445} e^{-0.166s}$$

$$C_{PI}(s) = k_p + \frac{k_i}{s}$$

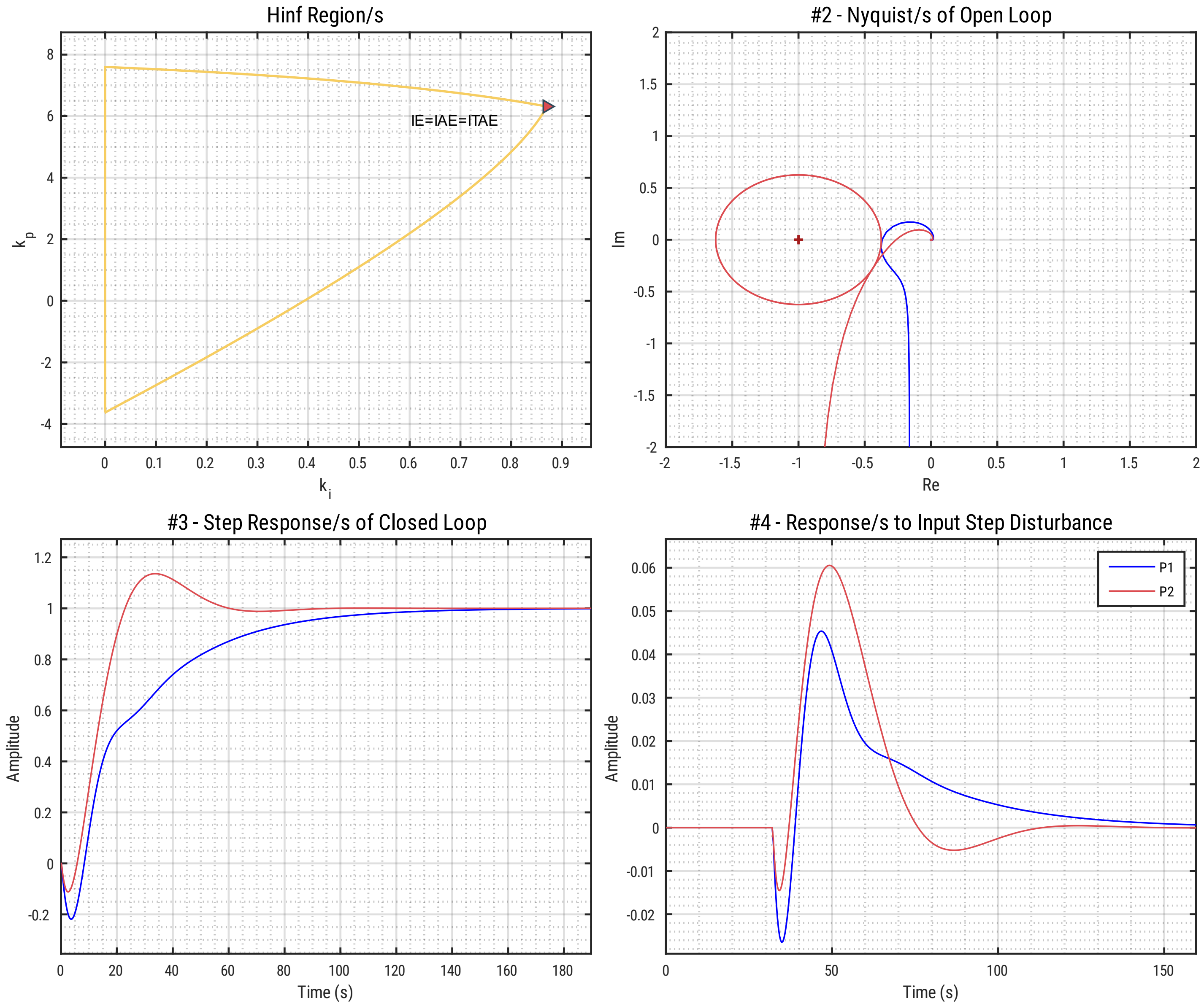
$$M_S = 1.6$$

Output:

$$IE: \quad k_p = 6.313, \quad k_i = 0.8704$$

$$IAE: \quad k_p = 6.313, \quad k_i = 0.8704$$

$$ITAE: \quad k_p = 6.313, \quad k_i = 0.8704$$



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Conclusion

- PID Hinf Designer is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE and H_∞ robustness constraints.
- PID Hinf Designer can be used for a wide range of proces models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, …) and also for so-called model sets created from any number of process transfer functions.
- PID Hinf Designer provide a new explicit algorithm to determine the H_∞- regions in the parameter plane of PI controller for all commonly used H_∞ limitations of the weighted sensitivity functions.
- PID Hinf Designer also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one frequency point). Moreover, the non-standard moment model set provided by the PIDMA-autotuner from the company REX Controls is also supported.
- Designing of PI(D) controller with typical specifications using PID Hinf Designer is a routine procedure that does not require deeper knowledge of control theory from the user.
- With more skills and efforts from the designer it should be possible to design high performance PID controllers extended with any linear compensator suitable (Resonant Controller, Smith predictor, Repetitive Control, …).

Appendix A: Isolation of H_∞-Region (1)

For more details see: Schlegel M., Medvecová P., Design of PI Controllers: H_∞ Region Approach. IFAC PapersOnLine 51-6 (2018), 13-17.

Proposition : If $C(s,k)=k_p+\dfrac{k_i}{s}$, $k=[k_p,k_i]$, $P(s)$ has no poles on the imaginary axis, and the design specification is

$$\|S(s,k)\|_\infty=\left\|\frac{1}{1+C(s,k)P(s)}\right\|_\infty=\left\|\frac{S_n(s,k)}{S_d(s,k)}\right\|_\infty\le\gamma\triangleq M_S\neq 1,$$

then the boundary of the H_∞-region K is contained in the solutions of the two systems of equations

(i) $S_n(j\omega,k)=0,\quad S_d(j\omega,k)=0,$

(ii) $|S(j\omega,k)|^2=\gamma^2,\quad \dfrac{\partial|S(j\omega,k)|^2}{\partial\omega}=0.$

The system of equations (i) has a solution $k_i=0$, i.e. any point on the axis k_p is a solution of this system. The solution of the system (ii) is determined by the parametric curves

Appendix A (2)

$$\left.\begin{aligned} k_i &= \frac{x_i\omega}{M_S}, \\ k_p &= \frac{x_i^2(A^2+B^2)^2 + x_i M_S(\omega(-A^2B_1 + 2AA_1B + B^2B_1) + B(A^2+B^2)) + \omega(M_S^2-1)(AA_1+BB_1)}{\omega M_S^2(2ABB_1 + A^2A_1 - A_1B^2)}, \end{aligned}\right\} \quad \omega\in[0,\infty),$$

where A, A_1, B, B_1 are the functions of ω defined by

$$P(j\omega) = A(\omega) + jB(\omega) \triangleq A + jB,$$

$$\frac{dP(j\omega)}{d\omega} = A_1(\omega) + jB_1(\omega) \triangleq A_1 + B_1,$$

and $x_i, i = 1,\ldots,l(\omega)$, $l(\omega)\in\{0,2,4\}$ are the frequency dependent real roots of quartic polynomial

$$a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e = 0$$

with the real frequency dependent coefficients

$$a = (A^2+B^2)^4,$$

$$b = 2M_S(A^2+B^2)^2(-\omega B_1A^2 + 2\omega BAA_1 + \omega B^2B_1 + BA^2 + B^3),$$

$$c = -(A^2+B^2)(2\omega A_1A^3 + 4\omega A^2BB_1M_S^2 + 2\omega A^2BB_1 - \omega^2A^2B_1^2M_S^2 - \omega^2A^2A_1^2M_S^2 - A^2B^2M_S^2 - 8\omega AA_1B^2M_S^2 + \\ + 2\omega AA_1B^2 - \omega^2A_1^2B^2M_S^2 - B^4M_S^2 + 2\omega B^3B_1 - \omega^2B^2B_1^2M_S^2 - 4\omega B^3B_1M_S^2),$$

$$d = -2\omega M_S(-\omega A_1^2B^3M_S^2 - \omega A^2A_1^2BM_S^2 - 2AA_1B^3M_S^2 - \omega A^2BB_1^2M_S^2 + A^2B^2B_1M_S^2 - B^4B_1M_S^2 - \omega B^3B_1^2M_S^2 + \\ + 3\omega AA_1B^2B_1 - \omega A^2BB_1^2 + AA_1B^3 + 2\omega A^2A_1^2B + \omega B^3B_1^2 - \omega A^3A_1B_1 + A^2B^2B_1 + A^3A_1B + B^4B_1),$$

$$e = -\omega^2(M_S^2-1)(-A_1^2B^2M_S^2 - B^2B_1^2M_S^2 + 2AA_1BB_1 + B^2B_1^2 + A^2A_1^2).$$

Appendix A (3)

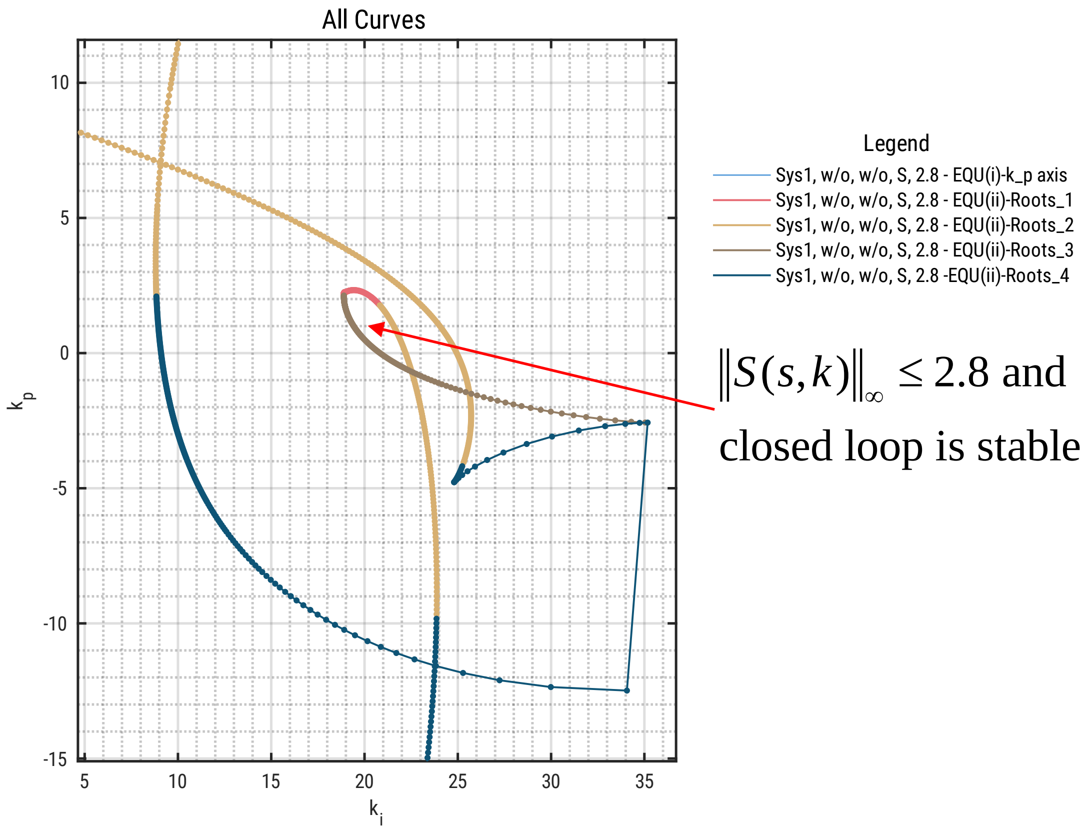
The curves representing the solutions of systems (i) and (ii) divide the parametric plane into regions. From them, it is necessary to select those that meet the design specifications. For this purpose, it is sufficient to test only one point of the respective region.

Example: $H_\infty -$ region for unstable process:

$$P(s)=\frac{s^3+4s^2-s+1}{s^5+2s^4+32s^3+14s^2-4s+50},$$

$$C(s,k)=k_p+\frac{k_i}{s}$$

$$\left\|S(s,k)\right\|_\infty=\left\|\frac{1}{1+C(s,k)P(s)}\right\|_\infty\leq M_S=2.8$$



Appendix A (4)

GUI:

Auxiliary Tools → Multiparametric Analysis

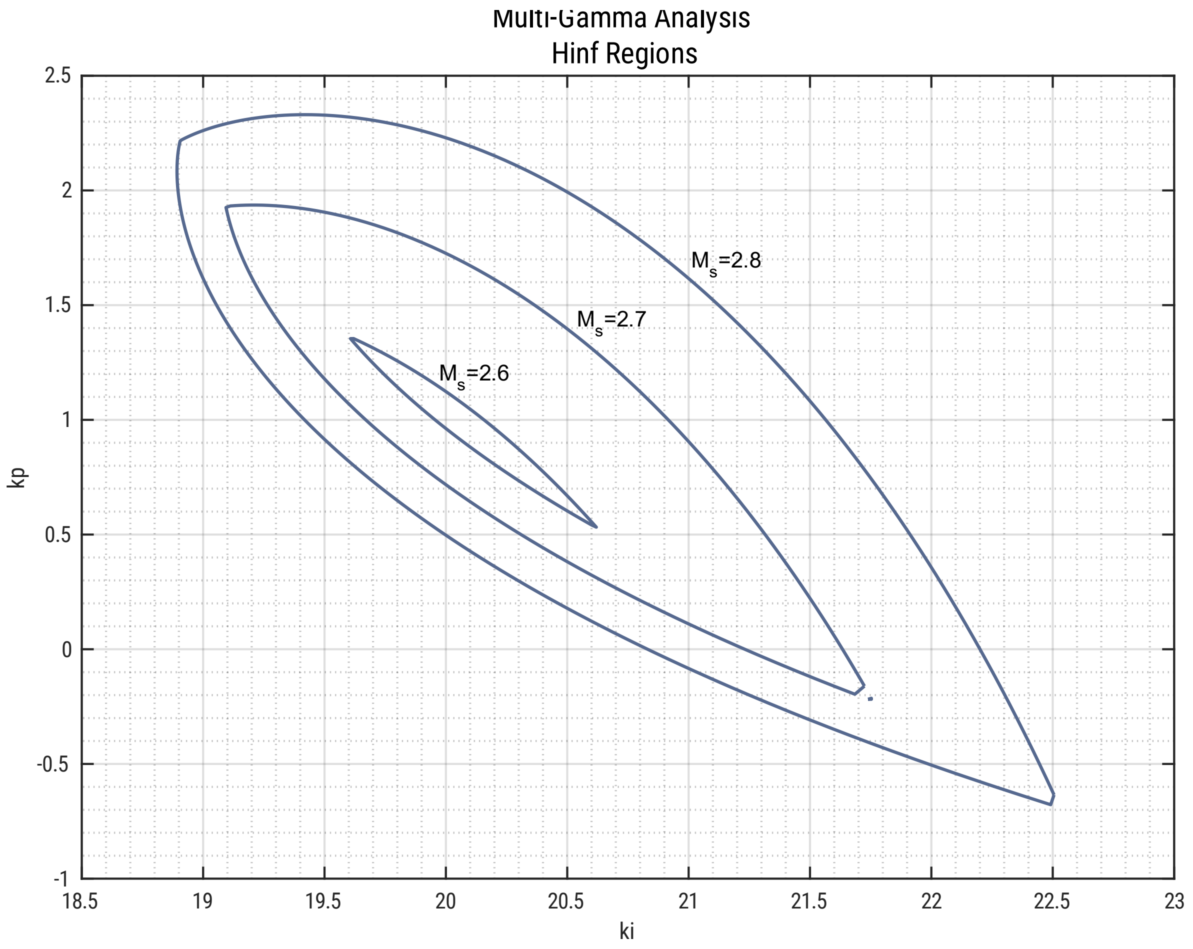
Example: $H_\infty -$ region for unstable process:

$$P(s)=\frac{s^3+4s^2-s+1}{s^5+2s^4+32s^3+14s^2-4s+50},$$

$$C(s,k)=k_p+\frac{k_i}{s}$$

$$\left\|S(s,k)\right\|_\infty=\left\|\frac{1}{1+C(s,k)P(s)}\right\|_\infty\le M_S$$

$$M_S\in\{2.6,2.7,2.8\}$$



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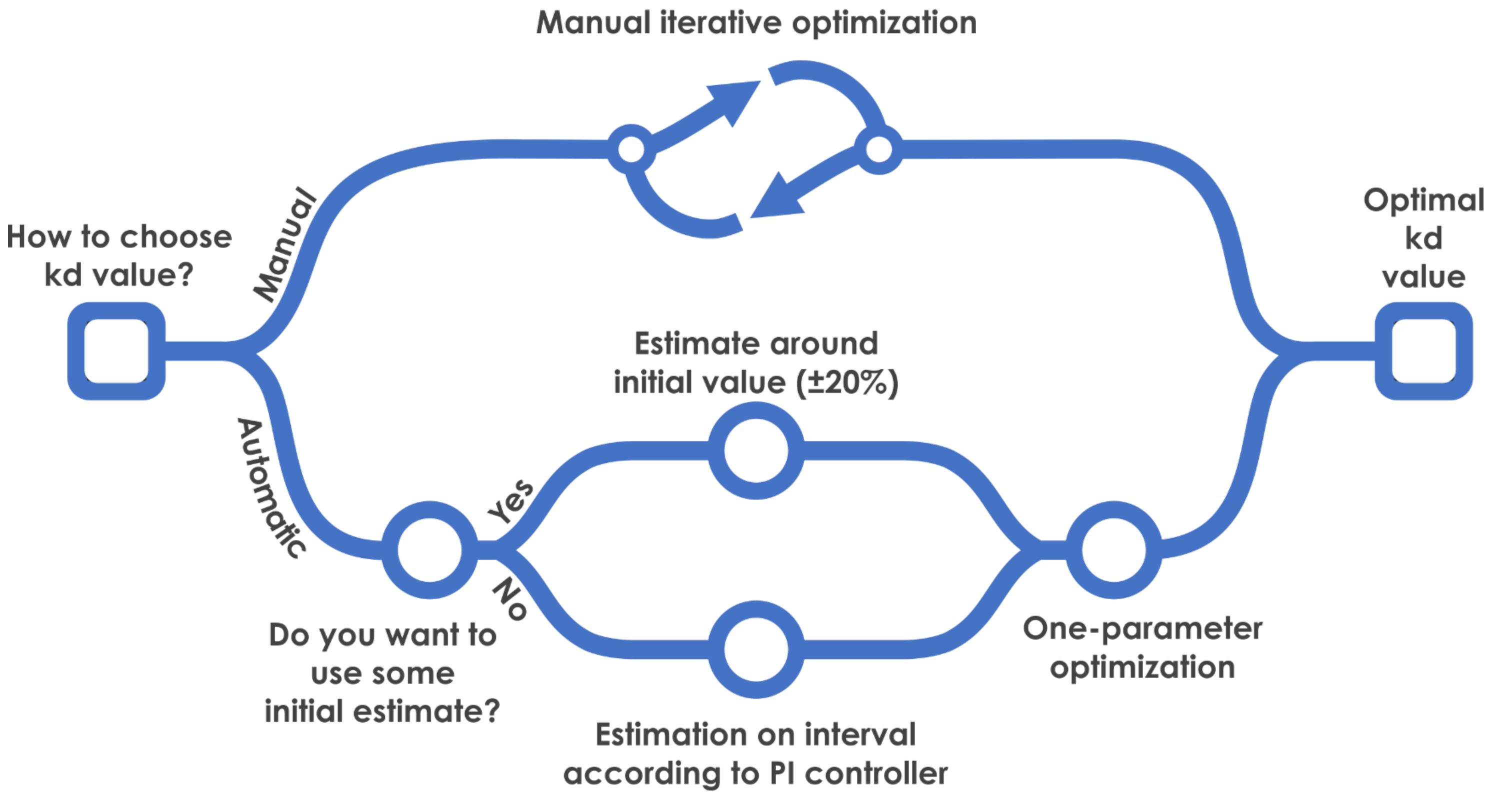
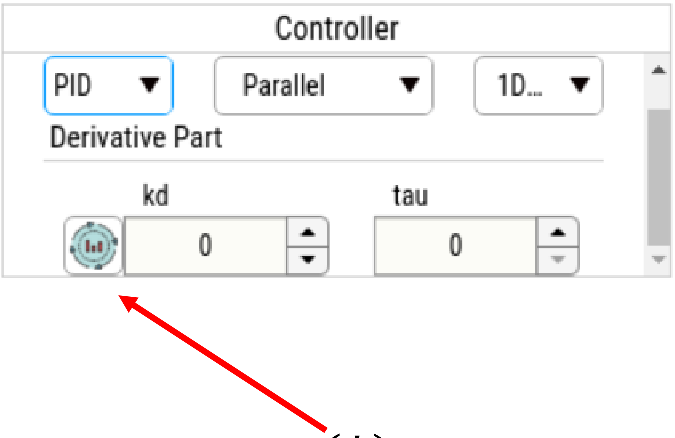
Appendix B: Selection of kd and tau

It is recommended to start with the ideal PID controller ($\tau = 0$). If there exists a PI controller for the given design specification with parameters k_p, k_i, ($T_i = k_p / k_i$), then it is recommended to estimate optimal k_d in the interval $\left[0.2k_p^2/k_i, 0.3k_p^2/k_i\right]$ manually or with the help of GUI build-in function (*).





(*)

Appendix G: Application Examples

Magnetic Levitation System

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{F_{em1}}{m} + \frac{F_{em2}}{m} + g$$

$$\dot{x}_3 = \frac{1}{f_i(x_1)}\left(k_i u_1 + c_i - x_3\right)$$

$$\dot{x}_4 = \frac{1}{f_i(x_d - x_1)}\left(k_i u_2 + c_i - x_4\right)$$

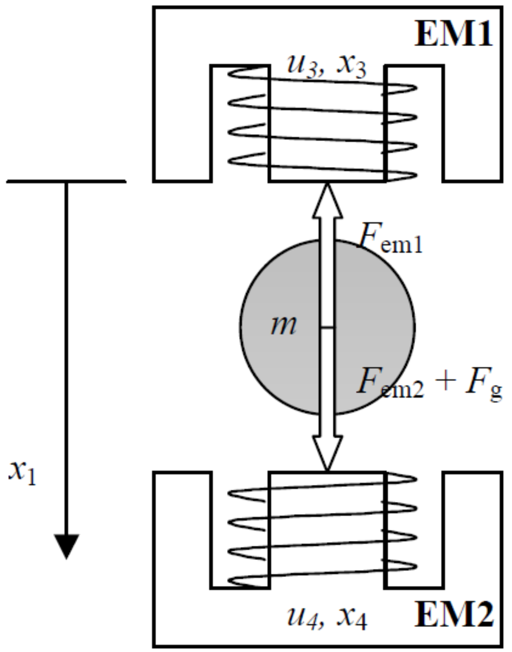
where

$$F_{em1} = x_3^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_1}{F_{emP2}}}$$

$$F_{em2} = x_4^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_d - x_1}{F_{emP2}}}$$

$$f_i(x) = \frac{f_{iP1}}{f_{iP2}} e^{-\frac{x}{f_{iP2}}}$$

F_{em1}—attraction force of the upper electromagnet [N],
F_{em2}—attraction force of the lower electromagnet [N],
F_g—force of gravity [N],
g—acceleration of gravity—9.81 [m/s^2]
m—mass of ball—0.0571 [kg],
u_1—electric voltage of the upper coil—<u_{min}, 1>,
u_{min} = 0.00498 [V],
u_2—electric voltage of the lower coil—<u_{min}, 1> [V],
x_d—distance between the magnets minus the ball diameter—defined by user [m],
x_1—distance from the upper magnet to ball —<0, 0.016> [m],
x_2—linear speed of the ball [m/s]
x_3—coil current of the upper electromagnet —<i_{min}, 2.38>,
i_{min} = 0.03884 [A],
x_4—coil current of the lower electromagnet —<i_{min}, 2.38> [A].



$c_i = 0.0242$ [A]
$F_{emP1} = 1.7521 \times 10^{-2}$ [H]
$F_{emP2} = 5.8231 \times 10^{-2}$ [H]
$f_{iP1} = 1.4142 \times 10^{-4}$ [ms]
$f_{iP2} = 4.5626 \times 10^{-3}$ [m]
$k_i = 2.5165$ [A]

Magnetic Levitation System: Linear Model Set

Transfer Functions from u1 to x1 (u2=0)

$$P_1(s) = \frac{-2.0893e4}{s^3 + 186.2891 \cdot s^2 - 1.6847e3 \cdot s - 3.1384e5}, \qquad (x_1 = 8[\mathrm{mm}])$$

$$P_2(s) = \frac{-2.7277e4}{s^3 + 288.7746 \cdot s^2 - 1.6847e3 \cdot s - 4.8649e5}, \qquad (x_1 = 10[\mathrm{mm}])$$

$$P_3(s) = \frac{-3.5611e4}{s^3 + 447.6417 \cdot s^2 - 1.6847e3 \cdot s - 7.5413e5}, \qquad (x_1 = 12[\mathrm{mm}])$$

[ML1] Hypiusová M., Kozáková A.: Robust PID Controller Design for the Magnetric Levitation System: Frequency Domain Approach. 21st International Conference on Process Control (PC), June 6-9, 2017, Štrbské Pleso, Slovakia

PID Hinf Designer

Input :

Model Set: $\{P_1, P_2, P_3\}$

Design specification:

2DOF PID controller

Setpoint tracking, IAE

$M_S \le 2.0, \;\; M_T \le 1.7$

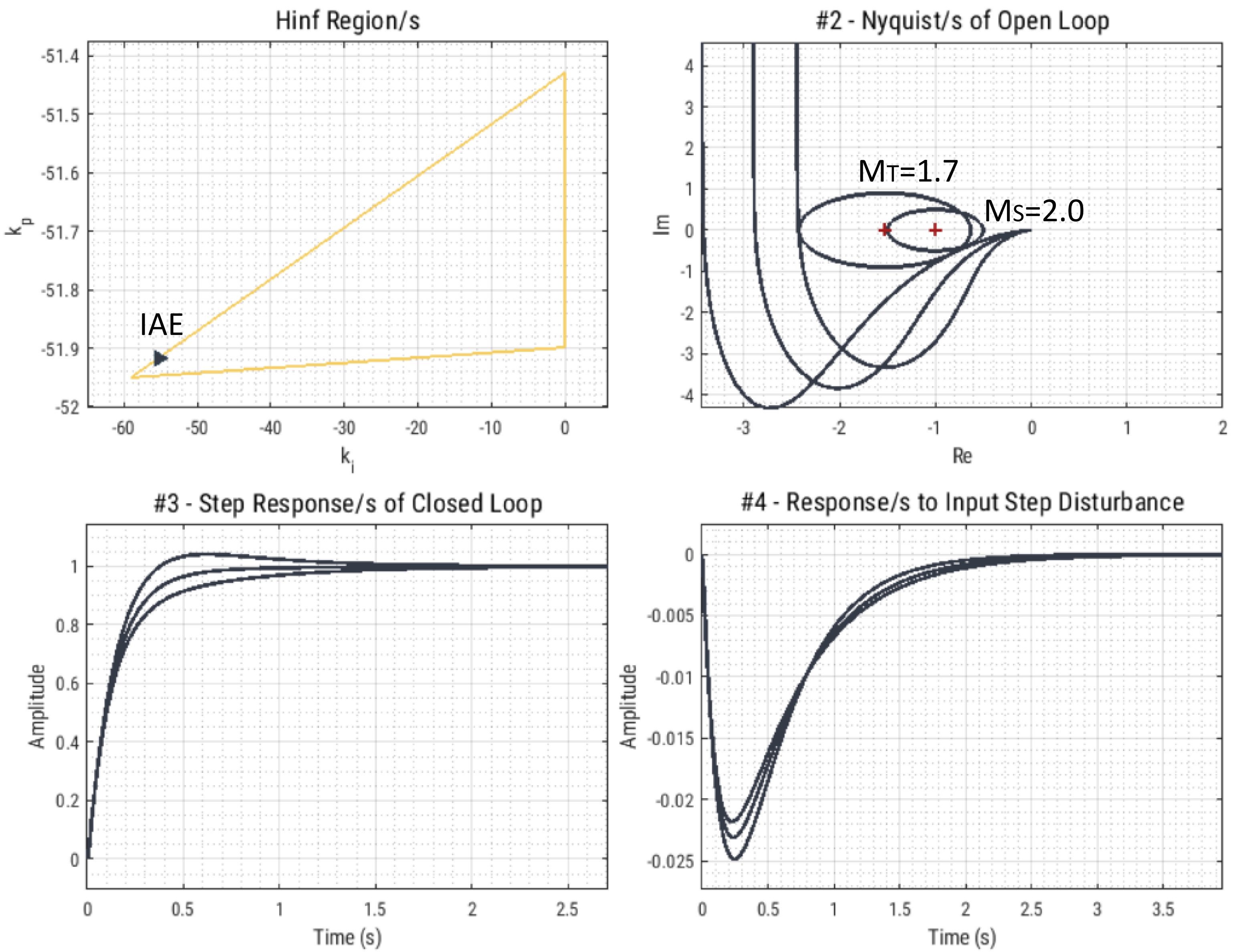
Output :

$k_p = -51.95$

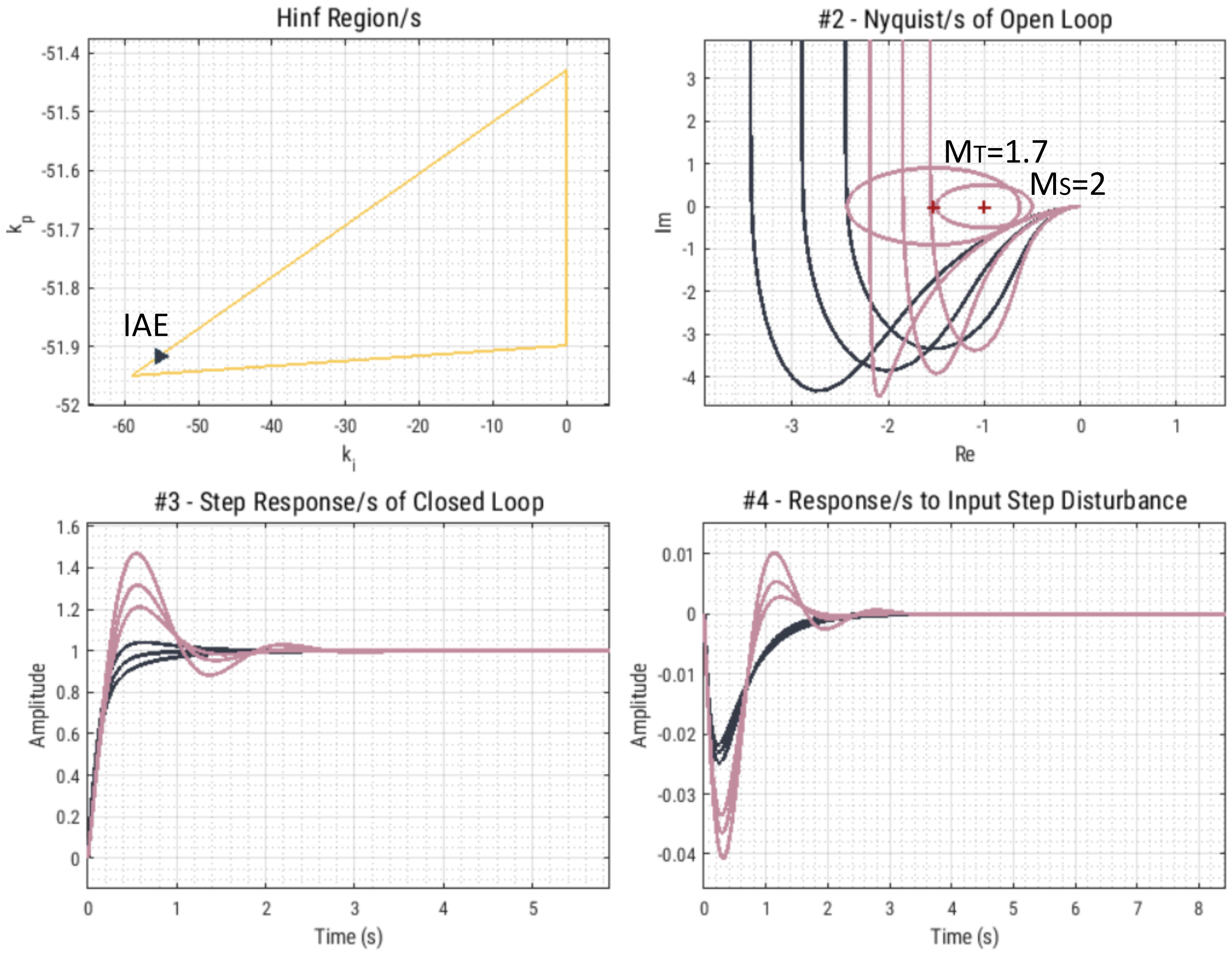
$k_i = -59.07$

$k_d = -3.63$

$b = 0.5, \;\; c = 0.0$



Comparison with the PID-controller proposed in [ML1]



PID Hinf:

2DOF PID - controller

$k_p = -51.95$

$k_i = -59.07$

$k_d = -3.63$

$b = 0.5,\ c = 0.0$

[ML1]:

1DOF PID - controller

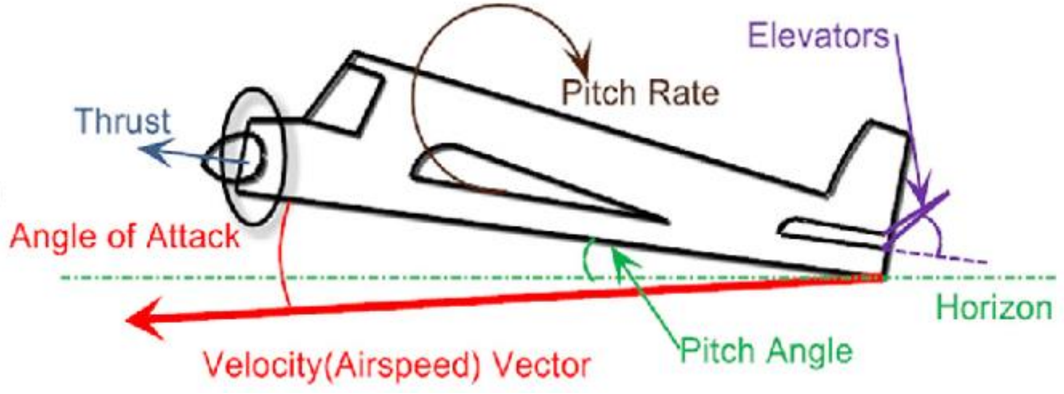
$k_p = -33.27$

$k_i = -61.04$

$k_d = -4.532$

Longitudinal motion of F4E fighter aircraft

We consider a model of the longitudinal motion of an F4E fighter aircraft [LM1], [LM2]. The input is the elevator position, the output is the pitch rate, and the system is linearized around four representative flight conditions:



$$P_i(s) \triangleq \frac{b^i(s)}{a^i(s)}, \quad i = 1, \dots 4.$$

Mach 0.5, 5000 ft: $a^1(s) = -52.75 + 22.00s + 15.84s^2 + s^3, \quad b^1(s) = -163.8 - 185.4s$

Mach 0.85, 5000 ft: $a^2(s) = -122.5 + 34.93s + 17.12s^2 + s^3, \quad b^2(s) = -789.1 - 507.8s$

Mach 0.9, 35000 ft: $a^3(s) = -14.64 + 17.51s + 15.33s^2 + s^3, \quad b^3(s) = -101.8 - 158.3s$

Mach 1.5, 35000 ft: $a^2(s) = 269.1 + 43.60s + 15.74s^2 + s^3, \quad b^4(s) = -251.4 - 304.2s$

[LM1] J. Ackermann. Robust Control Systems with Uncertain Physical Parameters. Springer Verlag, Berlin, 1993.

[LM2] Henrion D., Šebek M., Kučera V.: *Positive polynomials and robust stabilization with fixed – order controllers. IEEE Trans. Automatic Control AC – 48* (2003), 7.

PID Hinf Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PI controller

Setpoint tracking, IAE

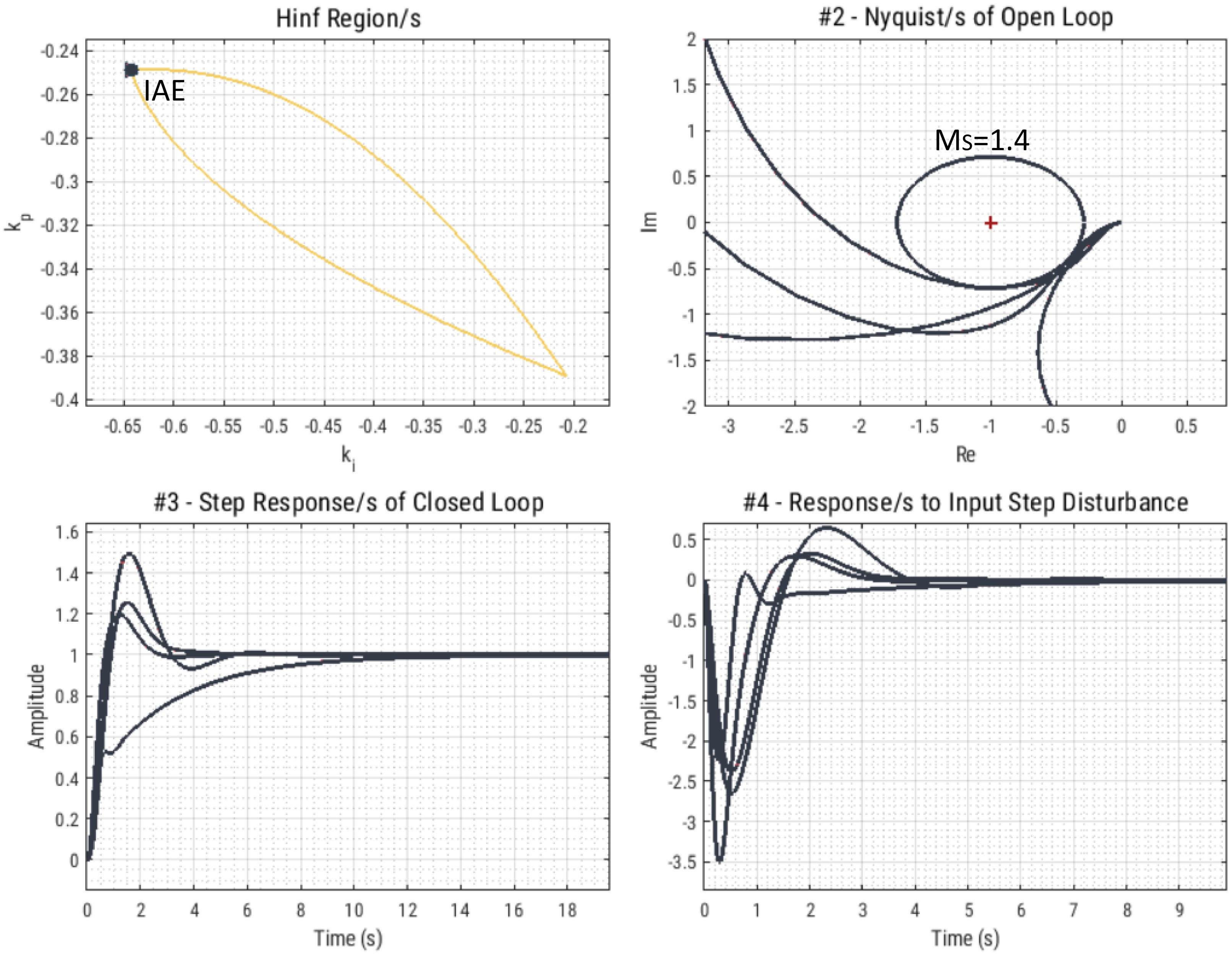
$M_S \leq 1.4$

Output :

$k_p = -0.25$

$k_i = -0.64$

$b = 0.0, \ c = 0.0$



PID Hinf Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$M_S \leq 1.4, \;\; M_T \leq 1.4$

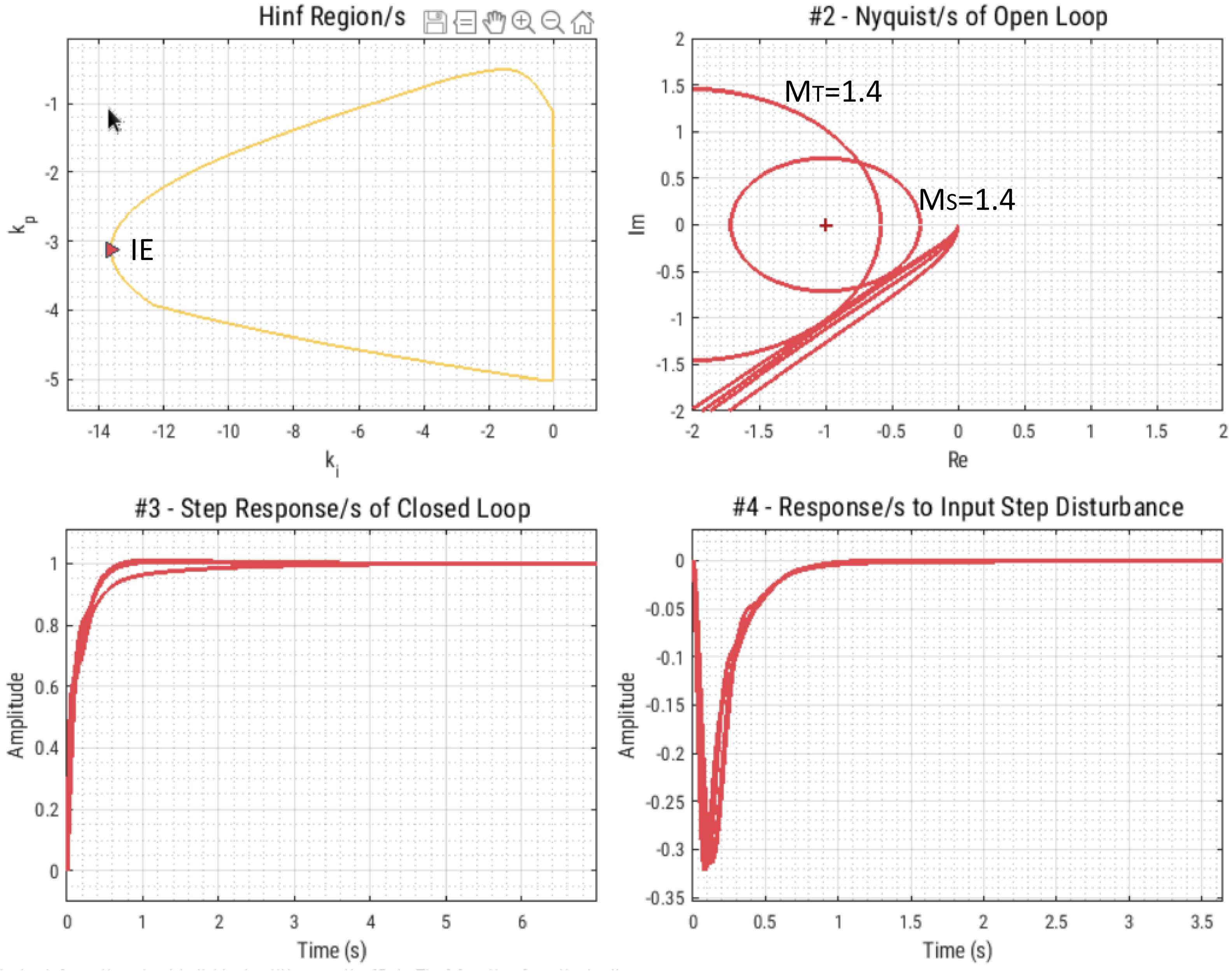
Output :

$k_p = -3.12$

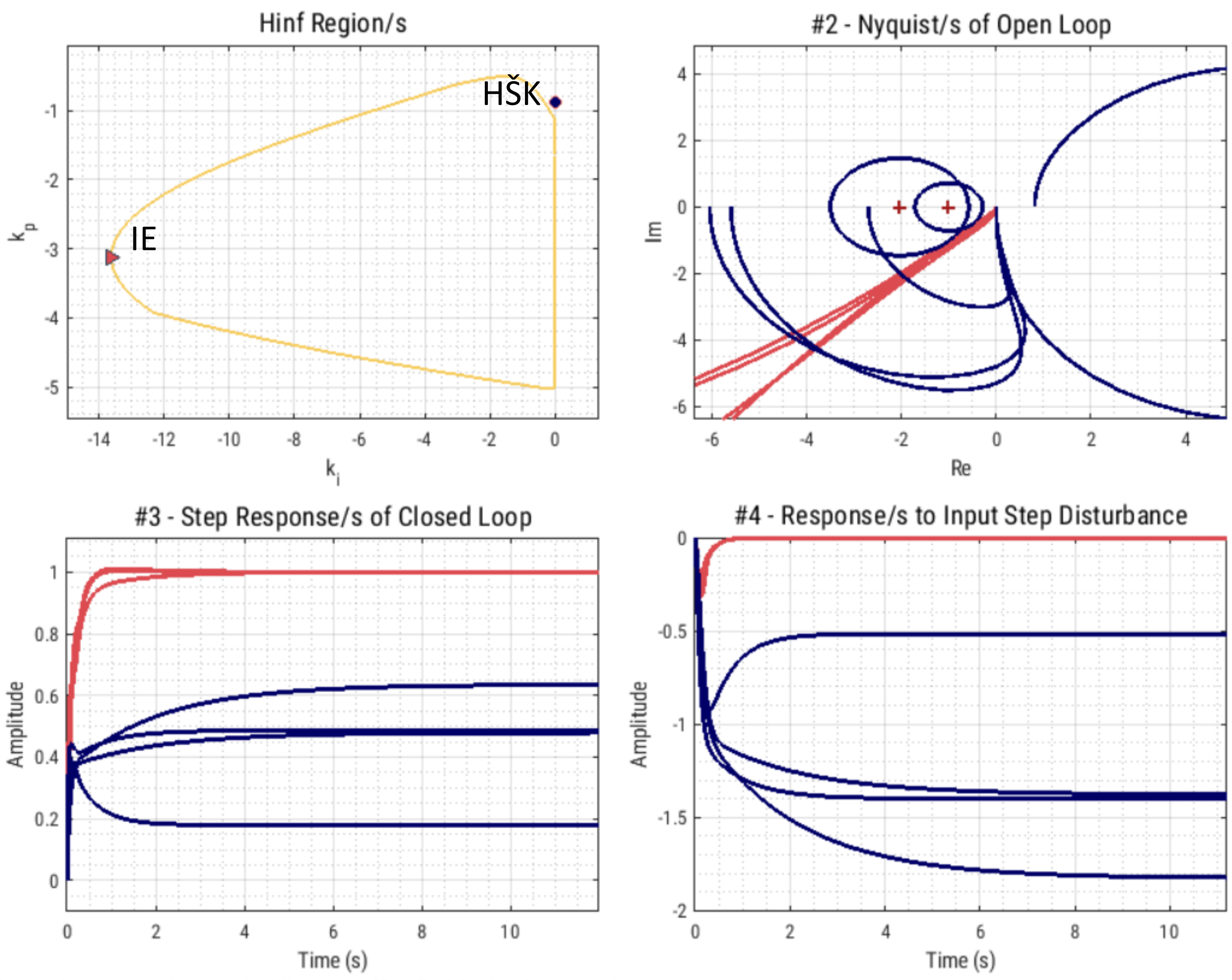
$k_i = -13.63$

$k_d = -0.06$

$b = 0.4, \;\; c = 0.6$



Comparison with the P-controller proposed in [LM2]



PID Hinf:

2DOF PID - controller

$k_p = -3.12$

$k_i = -13.63$

$k_d = -0.06$

$b = 0.4, \ c = 0.6$

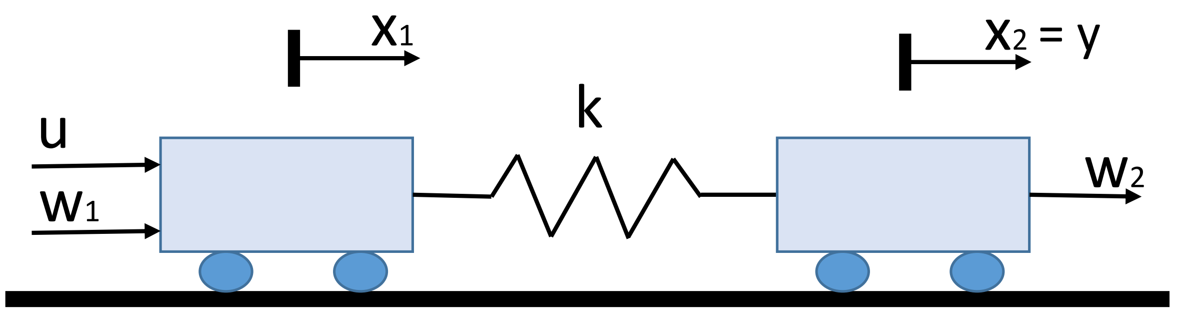
HŠK [LM2]:

P - controller

$k_p = -0.8698$

Benchmark Problem for Robust Control

Wie, B. and D.S. Bernstein (1990). A benchmark problem for robust control design. In: *Proc. American Control Conference*. San Diego, CA, USA. pp. 961–962.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k/m_1 & k/m_1 & 0 & 0\\ k/m_2 & -k/m_2 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1/m_1\\ 0\end{bmatrix}u+\begin{bmatrix}0 & 0\\ 0 & 0\\ 1/m_1 & 0\\ 0 & 1/m_2\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix}$$

$$P_{ux_2}(s,k)=\left\{\frac{k}{s^2(s^2+2k)}:k\in[0.5,2]\right\}\ \rightarrow\ P(s,k,\xi)=\frac{k}{s\left(s^2+2\xi\sqrt{2k}\cdot s+2k\right)},\quad \xi\rightarrow 0$$

$$\uparrow \qquad\qquad\qquad\qquad \downarrow$$

$$PD\ ({}^2k_p={}^1k_i,\ {}^2k_d={}^1k_p)\ \leftarrow\ PI\ ({}^1k_p,\ {}^1k_i)$$

PID Hinf Designer

Input :

Model Set: $\{P_1, P_2, P_3\}$

$P_1(s) = P(s, 0.5, 0.1),$

$P_2(s) = P(s, 1.0, 0.1),$

$P_3(s) = P(s, 2.0, 0.1).$

Design specification:

1DOF PI + compensator $F(s)$

$$F(s) = \left(\frac{\Omega^2}{\left(s^2 + 2\xi\Omega s + \Omega^2\right)} \right)^2$$

$\Omega = 0.9, \ \xi = 0.7$

Setpoint tracking, IE

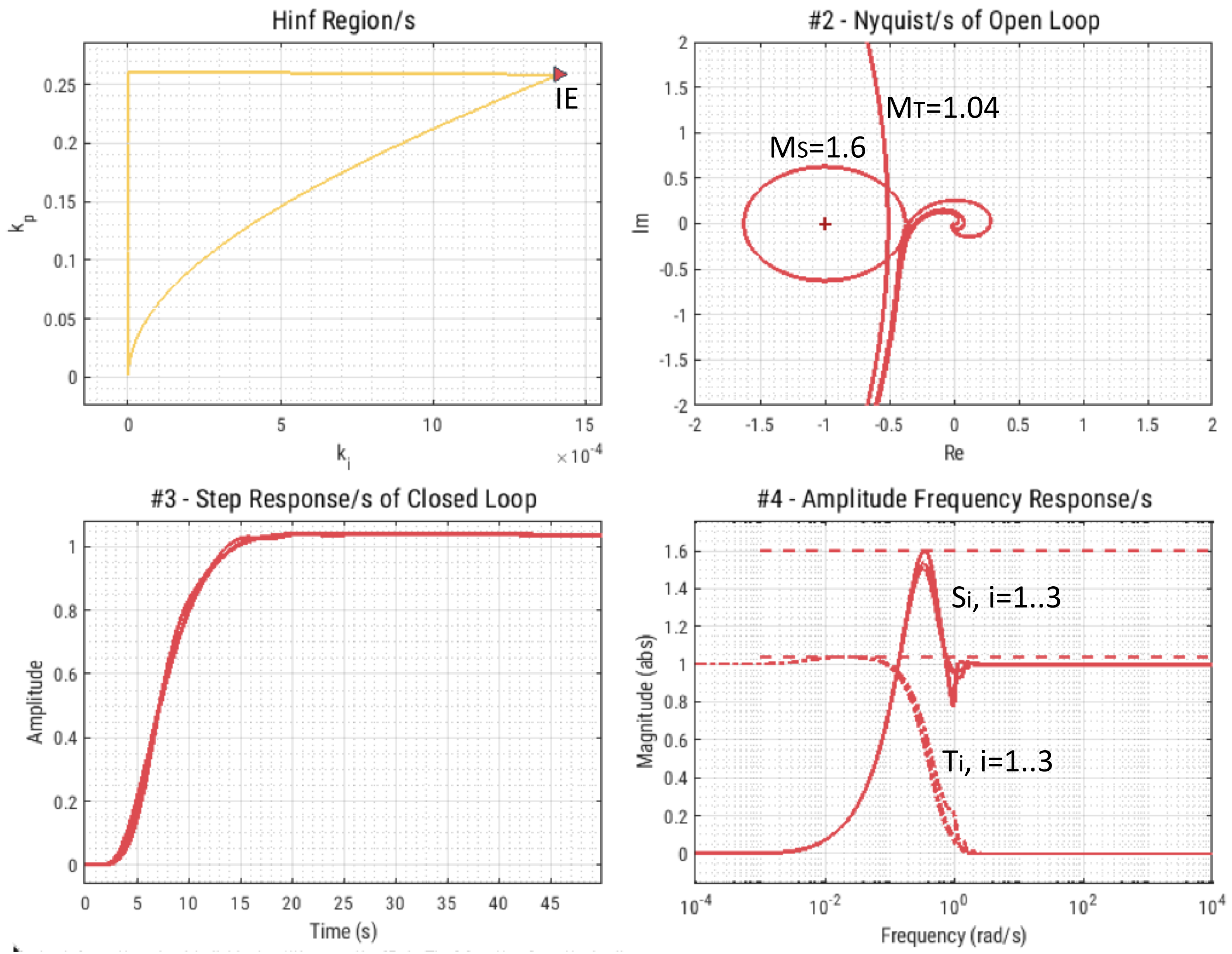
$M_S \leq 1.4, \ M_T \leq 1.05$

Output :

$k_p = 0.2586$

$k_i = 0.001413$

$b = 0.0$



PID Controller Design using One Frequency Point

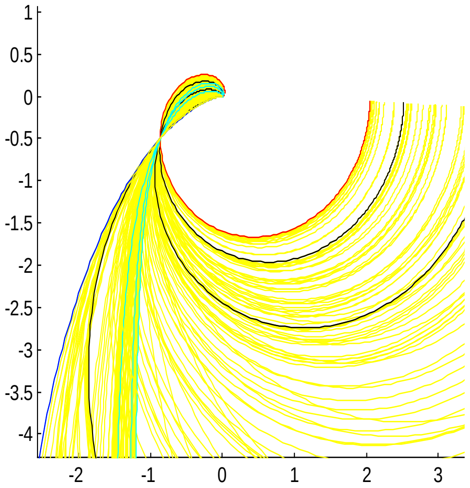
SCHLEGEL, M.: Nový přístup k robustnímu návrhu průmyslových regulátorů. Habilitační práce, Západočeská univerzita v Plzni, 2000.
https://www.schlegel.zcu.cz/downloads.php?lng=eng
SCHLEGEL, M.: Exact Revision of the Ziegler-Nichols Frequency Response Method. In Proceedings of the IASTED International Conference Control and Application, Cancun, Mexico, 2002, p. 121-126. ISBN 088986330X, ISSN 1025-8973 .

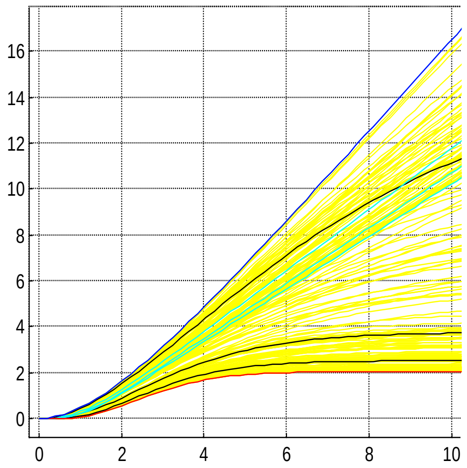
Definition (One Point Model Set). We are given one disturbance free sample of the plant frequency responce F_1, ω_1 and a fixed $n \in \{2,..,\infty\}$. A plant model $P(s)$ is an element of the plant family $S^n_{\mathbb{R}^-}(F_1, \omega_1)$ if it is consistent with the two following conditions:

(i) (A priori Hyposisis)

$$P(s) = \frac{1}{p(s)},$$

where $p(s)$, $\deg(p(s)) \leq n$, is a polynomial with real nonnegative coeficients, and all roots of $p(s)$ lie in the interval $(-\infty, 0]$.

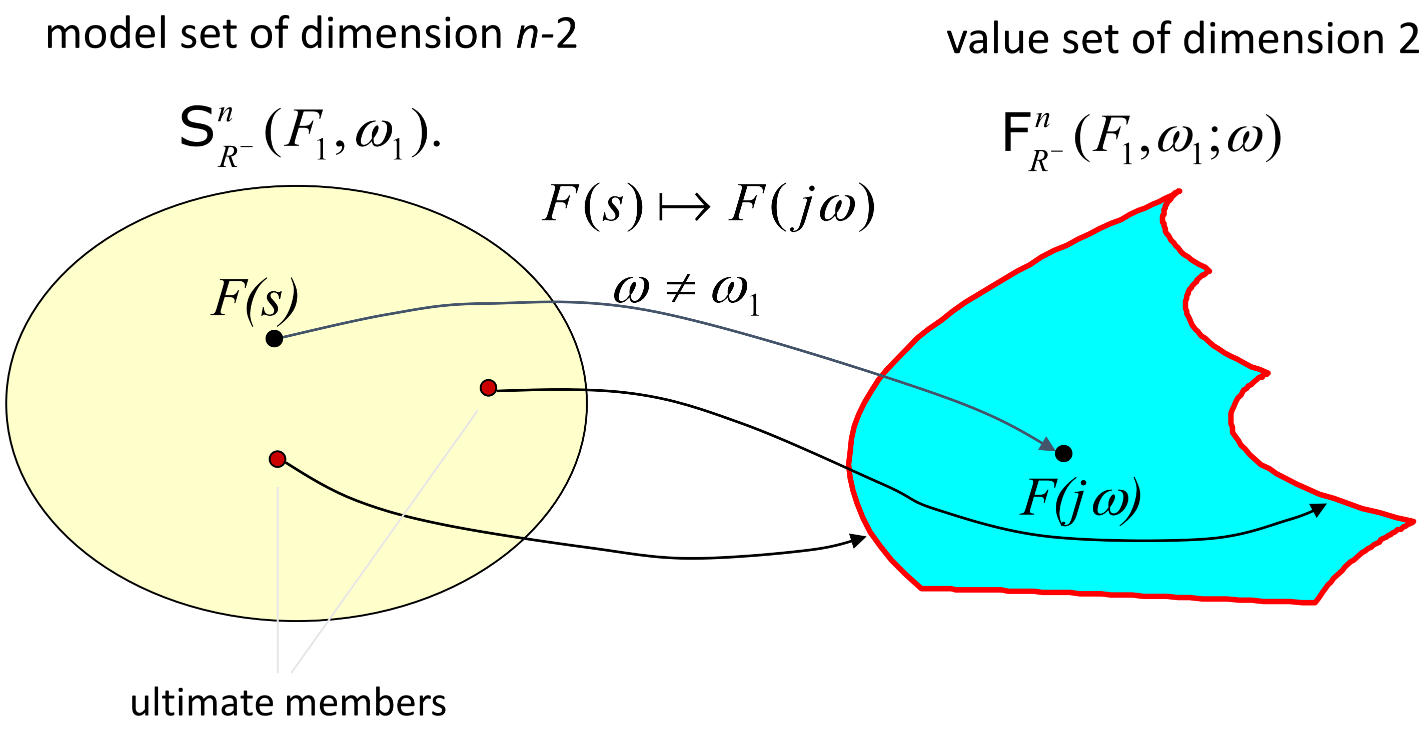




(ii) (Experimental Data Interpolation)

$$P(j\omega_1) = F_1, \quad -2\pi < \arg P(j\omega_1) \leq 0.$$

Main Idea of Solution



Only ultimate members of the unfalsified plant family can play an active role in the Nyquist curve constraints.

PID Hinf Designer

Input :

Model Set:

$S^{n}_{\mathbb{R}^-}(F_1, \omega_1)$,

$n = 10,\ F_1 = e^{-1.8j},\ \omega_1 = 1$

Design specification:

2DOF PI controller

Setpoint tracking, IE

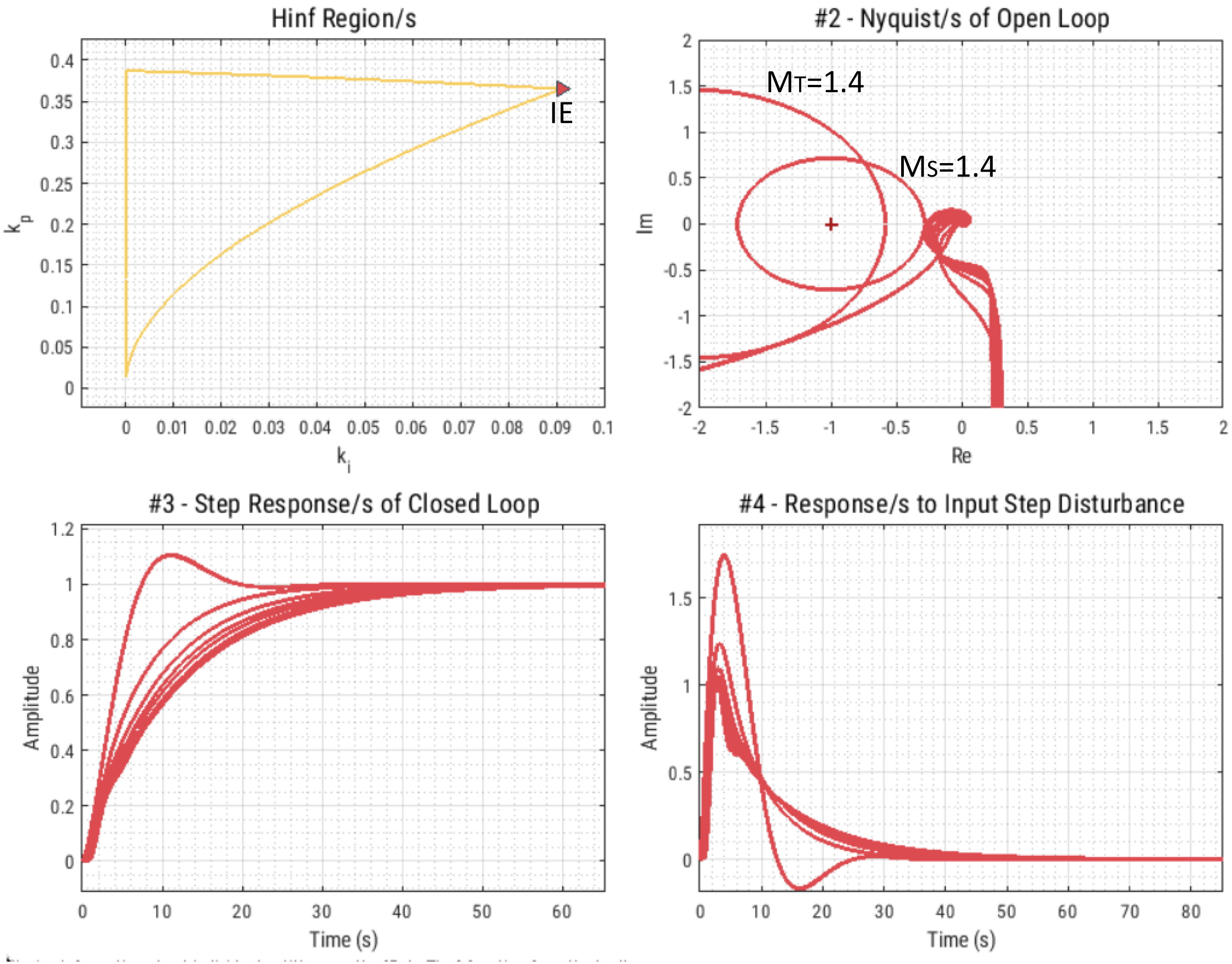
$M_S \le 1.4, M_T \le 1.4$

Output :

$k_p = 0.37$

$k_i = 0.091$

$b = 0.3$



PID-Autotuner PIDMA

SCHLEGEL M.: A new approach to the robust design of industrial controllers. Habilitation thesis, University of West Bohemia in Pilsen, 2000. (in Czech) https://www.schlegel.zcu.cz/downloads.php?lng=eng
SCHLEGEL M., Večerek O.: Robust design of Smith predictive controller for moment model set . *Proceedings of the 16th IFAC World Congress*, p. 427-432, Elsevier, Oxford, 2006.
SLAVÍČEK L., BALDA P., SCHLEGEL M.: Comparison of Siemens and REX Controls PI(D) Autotuners. 2021 23rd International Conference on Process Control (PC), June 1–4, 2021, Štrbske Pleso, Slovakia.
SCHLEGEL M., BALDA P., ŠTĚTINA M.. Robustní PID autotuner: momentová metoda. Automatizace, 46(4):242–246, 2003.

Definition ((κ,μ,σ^2) - Model Set). We are given the first three moments m_0, m_1, m_2 of the process impulse response $h(t)$ and fixed $n \in \{2,..,\infty\}$. A transfer function $P(s)$ is an element of the plant family $S^n_{\mathbb{R}^-}\left(\kappa,\mu,\sigma^2\right)$ if it is consistent with the two following conditions:

(i) (A priori Hyposisis)

$$P(s) = \frac{1}{p(s)},$$

where $p(s),\ \deg(p(s)) \le n$, is a polynomial with real nonnegative coeficients, and all roots of $p(s)$ lie in the interval $(-\infty, 0]$.

(*ii*) (Experimental Data)

$$m_i = \int_0^\infty t^i h(t)dt, \quad i = 0,1,2,$$

$$\kappa = m_0,$$

$$\mu = m_1/m_0,$$

$$\sigma^2 = m_2/m_1 - m_1^2/m_0^2.$$

PID Hinf Designer

Input :

Model Set:

$S^n_{\mathbb{R}^-}\left(\kappa,\mu,\sigma^2\right),$

$n=20,\ \kappa=1,\ \mu=1,\ \sigma^2=0.6$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$M_S \le 1.6, M_T \le 1.1$

Output :

$k_p = 2.587$

$k_i = 4.311$

$k_d = 0.25$

$b = 0.8,\ c = 1$