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PID H_∞ DESIGNER

INTRODUCTION



www.pidlab.com

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- The H_∞ control design problem
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PID design method

- For a long time, the development of PID controller design methods has been the goal of the control community. Despite that manual model-free tuning of controllers is still the most commonly used PID design method in industry.
- Tuning rules (Ziegler–Nichols, Lambda tuning, AMIGO method [1], Internal model control, Skogestad's SIMC method [2], ...)

Universal relations between model and controller parameters.

- Optimization-based method (MIGO [3], SWORD [4], MATLAB pidTuner)

Treats each process model individually.

[1] Astrom, K.J. and Hagglund, T.: *Advanced PID Control*. ISA, 2006, ISBN 1-55617-942-1

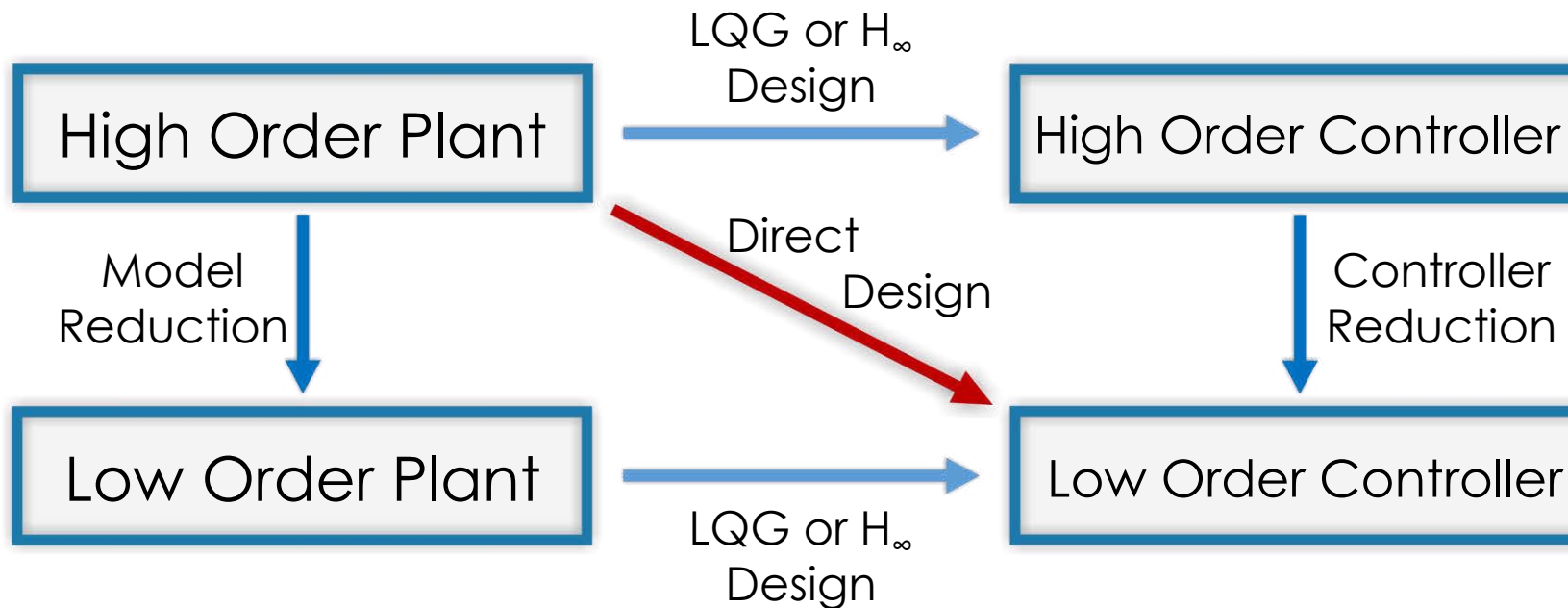
[2] Skogestad, S. and Grimholt, Ch.: *The SIMC Method for Smooth PIDController Tuning*. PIDControl in the Third Millennium.Springer. 2012

[3] Astrom, K.J., Panagopoulos, H., Hagglund, T.: *Design of PI Controllers based on Non-Convex Optimization*. Automatica, Vol. 34, No. 5, pp. 585–601, 1998.

[4] Garpinger, O.: *Analysis and Design of Software-Based Optimal PID Controllers*. PhD Thesis, Department of Automatic Control Lund University, 2015.

There exists no generally accepted design method for PID controller

The design procedures associated with modern control theory (H_∞ , LQG) provide high order controllers. Practice prefers simple controllers.



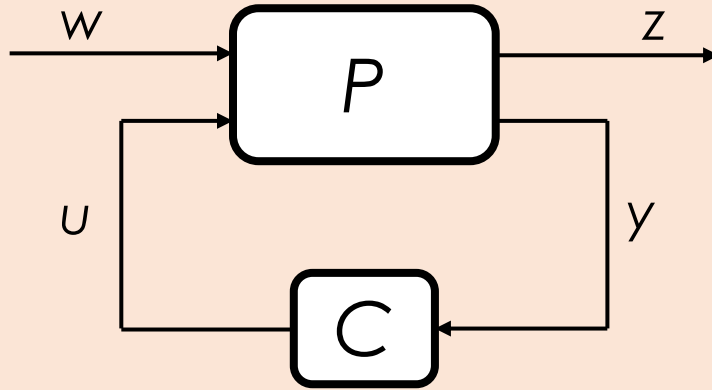
Anderson, B.D.O.: *Controller Design Moving from Theory to Practice*. 1992 Bode Prize Lecture.



Requirements for effective design method

- **Versatility:** It should be applicable to a wide range of systems (i.e. **stable/unstable/non minimal phase/oscillatory process transfer functions**)
- **Adaptability/Practicality** : It should have the possibility to introduce specifications that capture the essence of real control problems (i.e. **robustness/performance trade-off, servo/regulator problem**)
- **Clear answer:** The method should be robust in the sense that **it provides controller parameters if they exist**, or if the specifications cannot be meet an appropriate diagnosis should be presented

The general H_∞ Control Problem



minimize $\|H_{w \rightarrow z}(P, C)\|_\infty$
 subject to C stabilizes P internally
 $C \in \mathbf{C}$

$P = P(s)$ Given a real rational transfer matrix called the plant

$C \in \mathbf{C}$ Searched controller from the controller space \mathbf{C}

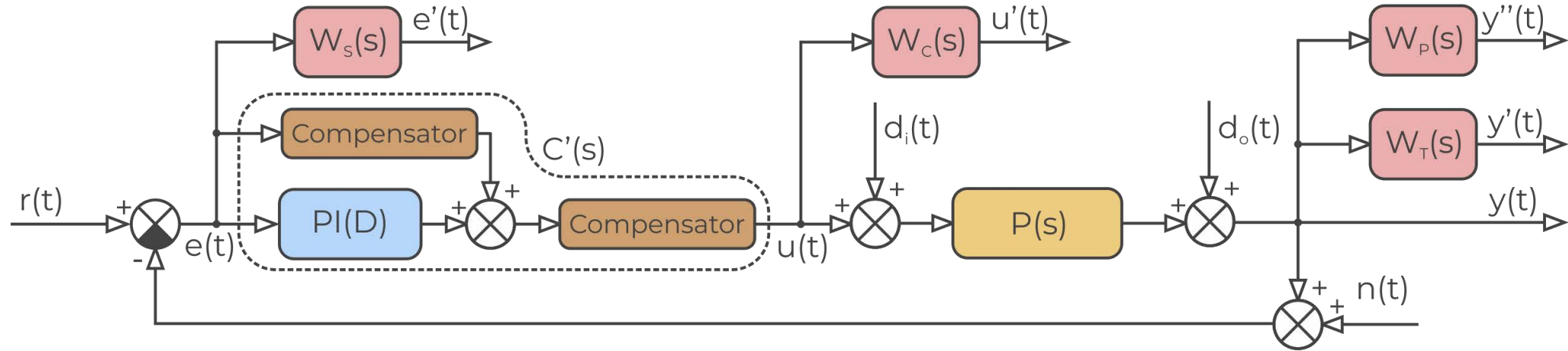
$H_{w \rightarrow z}(P, C)$ The closed-loop performance or robustness transfer matrix

In our considered case, $H \equiv H_{w \rightarrow z}(P, C)$ is a scalar function and it holds

$$\|H\|_\infty \triangleq \sup_{w \neq 0} \frac{\|Hw\|_2}{\|w\|_2} = \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2} = \max_{\omega} \bar{\sigma}(H(j\omega))$$

$$\|z\|_2 \triangleq \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} \text{Tr}(H(j\omega)H^H(j\omega)) d\omega \right)^{\frac{1}{2}}$$

The H_∞ Control Problem considered



Find all controllers C' for which it holds

$$\|H_{w \rightarrow z}(P, C')\|_\infty \leq \gamma$$

subject to C' stabilizes P internally

$$C' \triangleq (C_{PID} + C_{comp}) \cdot C_{comp} \in \mathbf{C}.$$

The performance or robustness

channel $H \equiv H_{w \rightarrow z}(P, C)$ is a scalar weighting closed-loop sensitivity function and it holds

$$\|H\|_\infty \triangleq \sup_{w \neq 0} \frac{\|Hw\|_2}{\|w\|_2} = \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2} = \max_{\omega} |H(j\omega)|$$

$$\|z\|_2 \triangleq \left(\int_{-\infty}^{+\infty} z^2(t) dt \right)^{\frac{1}{2}} = \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} |Z(j\omega)|^2 d\omega \right)^{\frac{1}{2}}$$



PID_{H_{inf}} Designer

Version: 3.2.0

Home Approach UI

The design of the controller and control loop is a crucial aspect of automation, which is aimed at managing diverse technological processes and equipment. The proper configuration of these control elements significantly impacts the efficiency and effectiveness of the resulting system. The purpose of this application is to find such a suitable configuration.

[Tutorial](#)

[Get Started >>](#)

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PID H_∞ Designer (1)

- **PID H_∞ Designer** is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE, ISE and H_∞ robustness constraints.
- **PID H_∞ Designer** can be used for a wide range of process models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, ...) and also for so-called model sets created from any number of process transfer functions.
- Supported design specifications reflect the essence of real control problems. Optimization of integral criteria IE, ISE, IAE, ITAE under H_∞ constraints is supported for both **load disturbance attenuation** and **set-point tracking problems**).
- Designing of PI(D) controller with typical specifications using **PID H_∞ Designer** is a routine procedure that does not require deeper knowledge of control theory from the user.

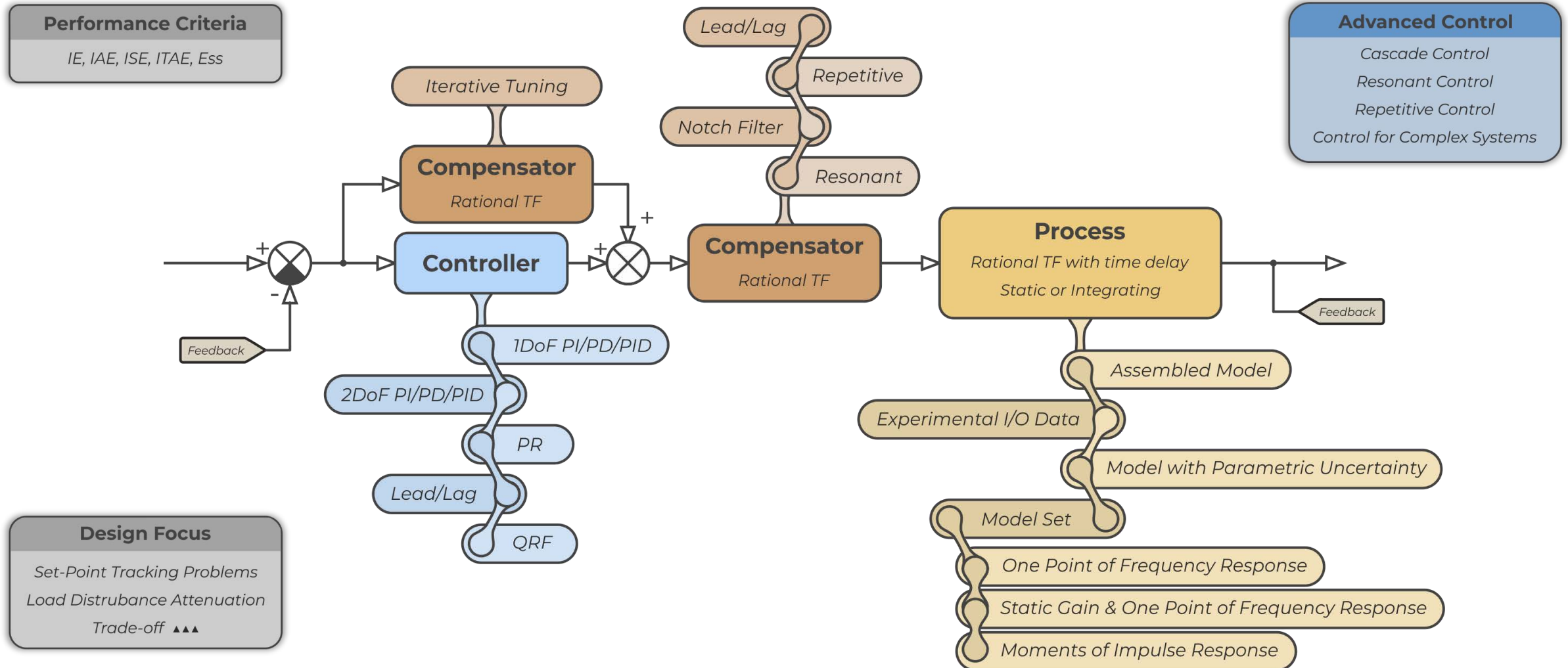


PID H_∞ Designer (2)

- With more skills and efforts from the designer it should be possible using **PID H_∞ Designer** to design high performance PID controllers extended with a suitable linear compensator (Cascade Controller, Resonant Controller, Smith Predictor, Repetitive Control, ...).
- **PID H_∞ Designer** also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one point of the frequency response). Moreover, the non-standard moment model set provided by the **PIDMA-autotuner** from the company **REX Controls** is also supported.

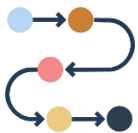


PID H_∞ Designer: Options



PID H_∞ Designer: Design Environment

The user can choose between two design environments. Each of them is specifically designed to accommodate users with different levels of expertise.



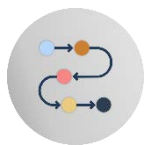
Step By Step

The environment is intended primarily for beginners who are working with the tool for the first time. For these reasons, the design process is divided into several steps (slides). All necessary information is then explained in the individual phases of the design process.

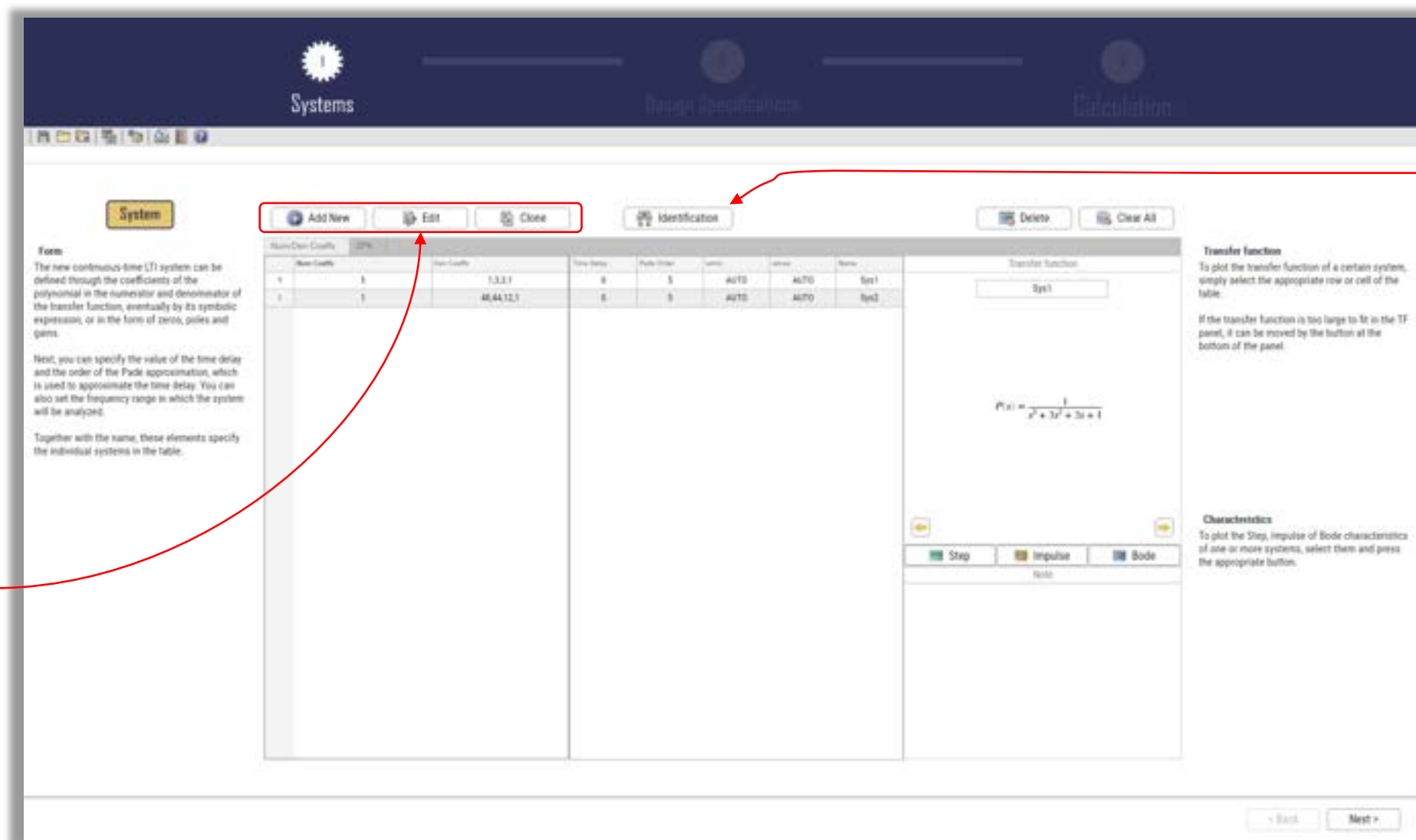


WorkSpace

The environment is more suitable for advanced users who are already familiar with the design process. This environment also includes a set of auxiliary functions and settings to streamline the design and analysis of the solution.



PID H_{∞} Designer GUI: Step By Step (1)



System

Form

The new continuous-time (CT) system can be defined through the coefficients of the polynomial in the numerator and denominator of the transfer function, eventually by its symbolic expression, or in the form of zeros, poles and gains.

Next, you can specify the value of the time delay and the order of the Padé approximation, which is used to approximate the time delay. You can also set the frequency range in which the system will be analyzed.

Together with the name, these elements specify the individual systems in the table.

Numerator Coeffs		Denominator Coeffs		Time Delay	Pole Order	Zeros	Gain	Name
1	1	1	3.337	0	0	AUTO	AUTO	Sys1
1	1	1	46.4412,1	0	0	AUTO	AUTO	Sys2

Identification

Transfer function

To plot the transfer function of a certain system, simply select the appropriate row or cell of the table.

If the transfer function is too large to fit in the TF panel, it can be moved by the button at the bottom of the panel.

Characteristics

To plot the Step, impulse or Bode characteristics of one or more systems, select them and press the appropriate button.

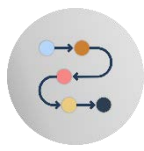
Step Impulse Bode

Plot

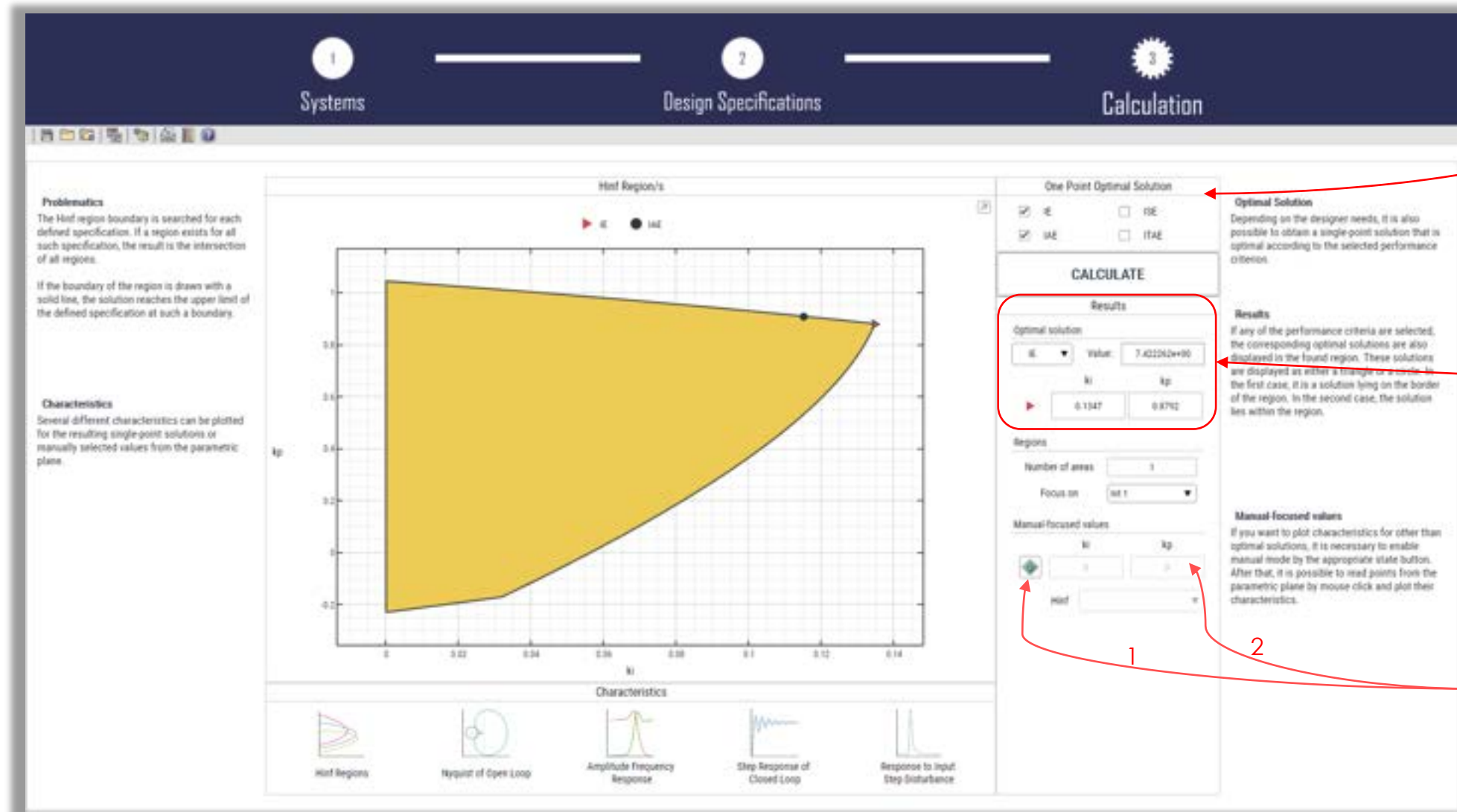
System
Identification
Experimental
I/O data
(See
[Appendix E](#))

Entering rational
transfer function
+ time delay of
processes





PID H_∞ Designer GUI: Step By Step (3)



Selection of the criterion function

The resulting controller

Manual tuning of the controller



PID H_∞ Designer GUI: WorkSpace



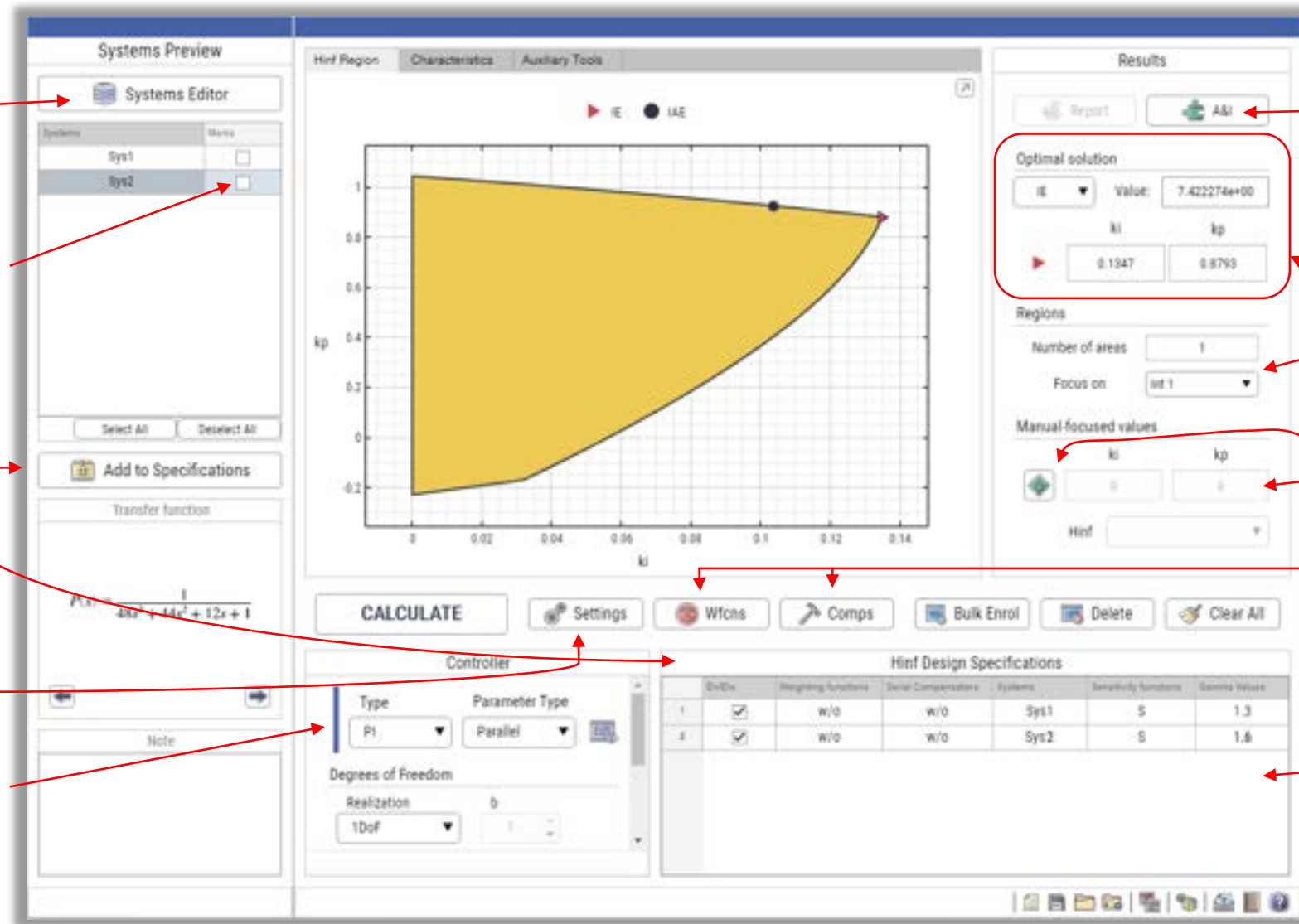
Entering transfer functions of processes

Selection of a model set

Enter H_∞ limitations

Selection of the design criterion

Select the controller type



Create a closed-loop assembled transfer function (e.g. for cascade control) or store actual controller for iterative modification via parallel compensator

The resulting controller

H_∞ region selection

Manual tuning of the controller

Enter weighting functions and compensators

Select weighting functions, compensators, systems, sensitivity functions and values of H_∞ limitations



PID H_∞ Designer GUI: WorkSpace

Controller Settings

Estimate kd value

Select the controller type

Show controller form

Manage DoF

Choose compensators

Affine-structured controller labelled as QRF → Conclusion (2)

The image displays two screenshots of the PID H_∞ Designer GUI, specifically the Controller settings window. The left screenshot shows the PID controller configuration, while the right screenshot shows the QRF (Affine-structured) controller configuration. Red arrows point from text labels to specific GUI elements in both screenshots.

Left Screenshot (PID Controller):

- Type:** PID (dropdown menu)
- Parameter Type:** Parallel (dropdown menu)
- Derivative part:** kd (input field, value 0), tau (input field, value 0)
- Degrees of Freedom:** Realization (dropdown menu, value 1DoF), b (input field, value 1), c (input field, value 1), Overshoot [%] (input field)
- Compensators:** Serial (dropdown menu, value w/o), Parallel (dropdown menu, value w/o)

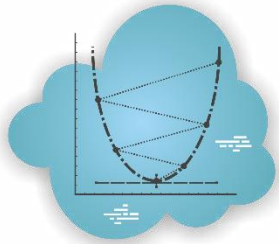
Right Screenshot (QRF Controller):

- Type:** QRF (dropdown menu)
- $R(s)$:** (input field)
- $F(s)$:** w/o (dropdown menu)
- Compensators:** Serial (dropdown menu, value w/o), Parallel (dropdown menu, value w/o)



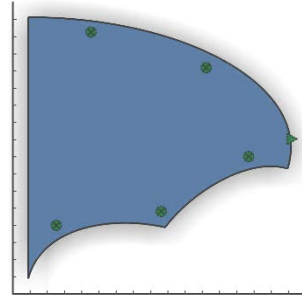
PID H_∞ Designer GUI: WorkSpace

Auxiliary Tools (1)



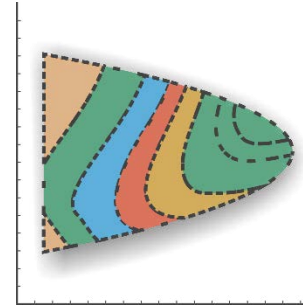
Find the Minimum Gamma Value

Finding the minimum gamma value for selected design constraints



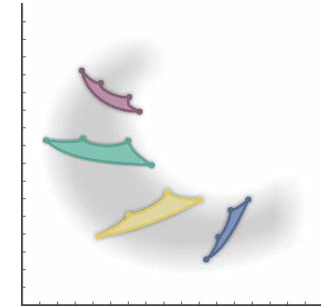
Multipoint Analysis

Analysis of the obtained optimal solutions with possibly of exploration over the H_∞ region to achieve the desired behavior



Performance Criteria Contour Line

Computing contour lines of selected criterion function for solutions from the H_∞ region



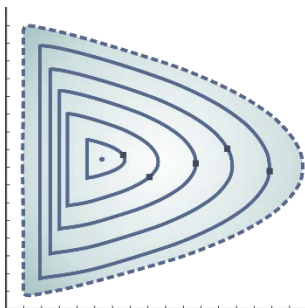
Open Loop-Value Set Region/s

Show open loop value set/s for selected frequencies. Value set/s represents a model uncertainty on these frequencies



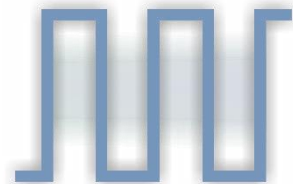
PID H_∞ Designer GUI: WorkSpace

Auxiliary Tools (2)



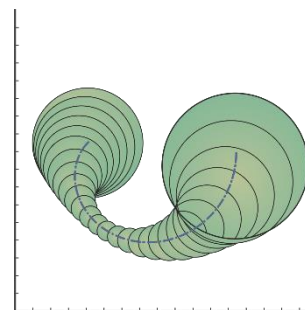
Multiparametric Analysis

Analysis of the choice of the parameter gamma, k_d , or τ according to several characteristics



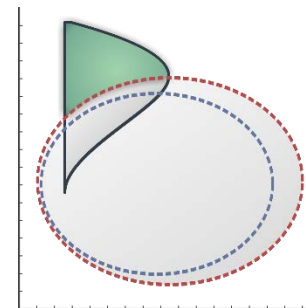
Signal Response

Simulate system responses for various types of signals.



Tolerance Circles

Calculation of tolerance circles of nominal open loop frequency characteristic in complex plane according to selected nominal gamma value and maximal gamma value



ϵ – Constraint

Find the solution that satisfy selected amplitude limit ϵ on the frequency interval between ω_1 and ω_2

PID H_∞ Designer GUI – Systems Editor

Parameter Uncertainty Model Set (See [Appendix C](#))

Rational Transfer Function + Time Delay

Experimentally Determined Model Set (See [Appendix D](#))

System Identification Experimental I/O data (See [Appendix E](#))

	Num Coeffs	Den Coeffs	Time Delay	Pole Order	zeros	poles	Name
1	1	1, 3, 3, 1	0	5	AUTO	AUTO	Sys1
2	1	48, 44, 12, 1	0	5	AUTO	AUTO	Sys2

Transfer function

Sys2

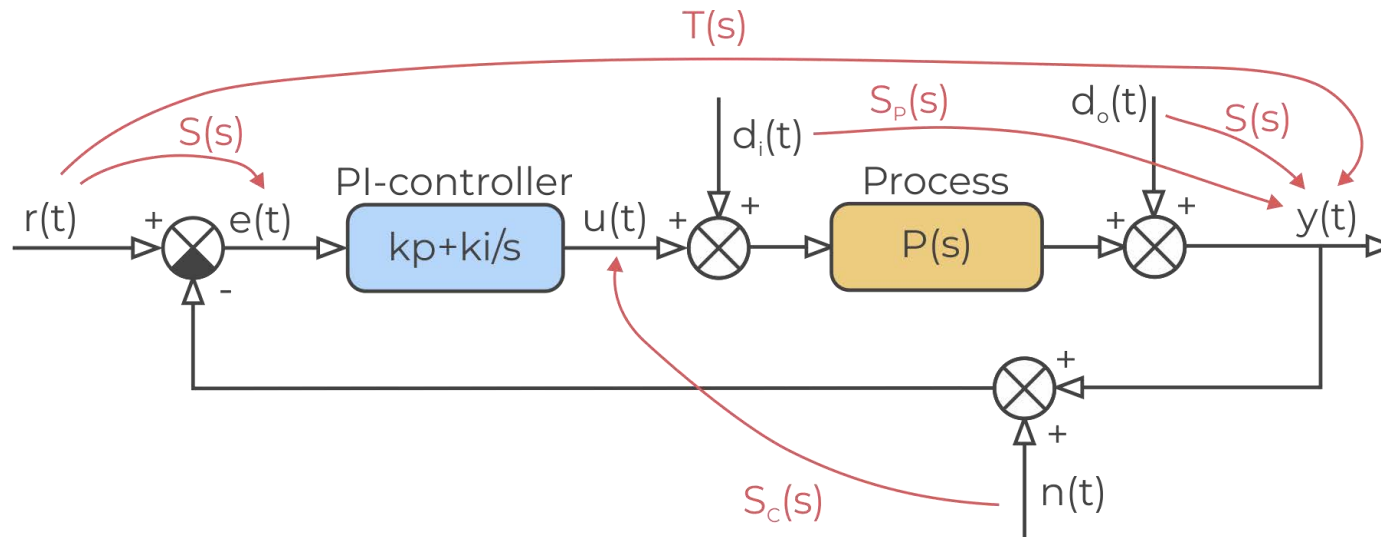
$$P(s) = \frac{1}{48s^3 + 44s^2 + 12s + 1}$$

Step Impulse Bode

Note

Return

Parameter Plane Formulation of Basic PI-Controller Design Problem



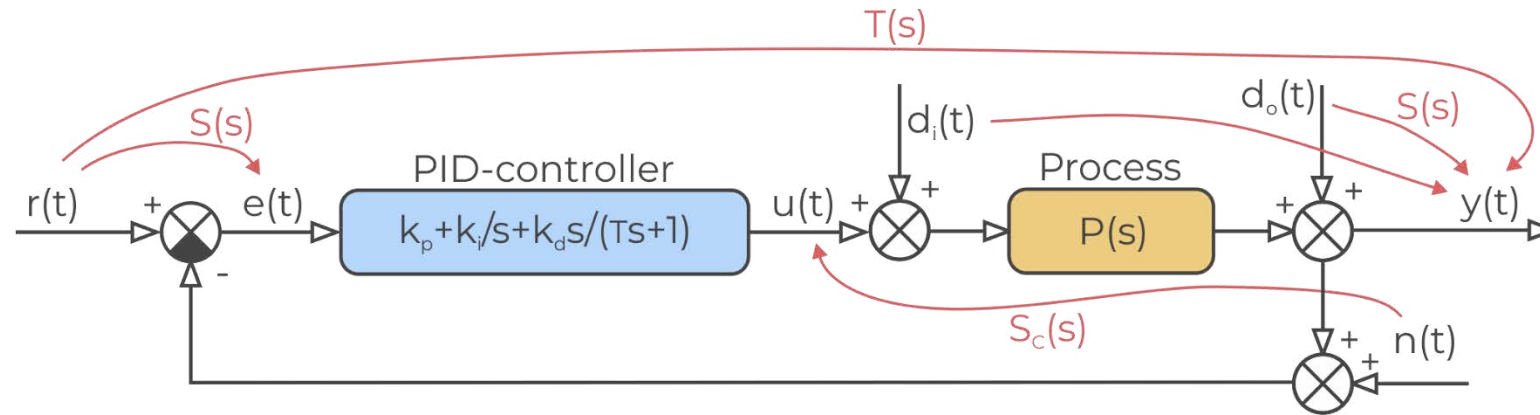
$H(s, k = [k_i, k_p]) \triangleq W(s)S_*(s, k)$, $S_* \in \{S, T, S_c, S_p\}$ weighting sensitivity function

$K = \left\{ [k_i, k_p] : \left\| \begin{pmatrix} S \\ T \\ S_c \\ S_p \end{pmatrix} \right\|_{\infty} \sup_{\omega} \left| \begin{pmatrix} S \\ T \\ S_c \\ S_p \end{pmatrix} (j\omega) \right| \leq \gamma, \text{ the closed-loop is stable} \right\}$ H_{∞} -region in the parameter plane

1) Find the H_{∞} -region K in the k_i - k_p plane. (See [Appendix A](#) for details.)

2) Find the optimal PI-controller in the H_{∞} -region K with respect to the criterion $IAE \triangleq \int_0^{\infty} |e(t)| dt$ for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

Parameter Plane Formulation of Basic PID-Controller Design Problem

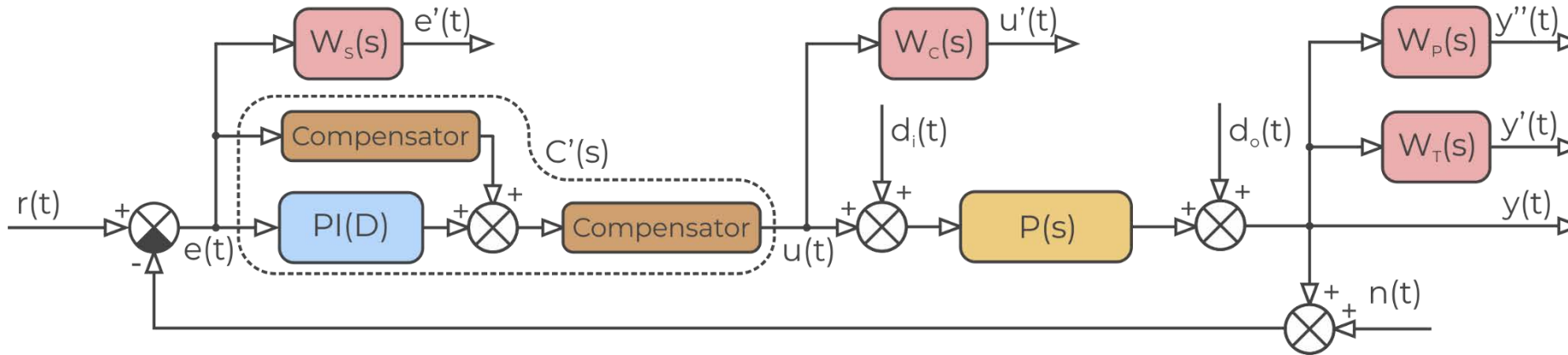


$H(s, k = [k_p, k_i, k_d, \tau]) \triangleq W(s)S_*(s, k)$, $S_* \in \{S, T, S_C, S_P\}$ weighting sensitivity function

$K_{[d, \tau]} \triangleq \left\{ \left[\begin{array}{c} k_p \\ k_i \\ k_d \end{array} \right] \parallel \left\| \right\|_{\infty} \triangleq \omega \mid \omega \mid \leq \gamma \text{ the closed-loop is stable} \right\} \dots \infty$ region in the parameter plane
for the fixed k_d and τ

- 1) Choose the derivative gain k_d and the time constant τ manually or with the help of a built-in function. (See [Appendix B](#) for details.)
- 2) Find the H_{∞} - region $K_{[k_d, \tau]}$ in the $k_i - k_p$ plane.
- 3) Find the optimal PID-controller in the H_{∞} region $K_{[k_d, \tau]}$ with respect to the criterion $IAE \triangleq \int_0^{\infty} |e(t)| dt$ for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

H_∞ limitations supported



$$\|H(s)\|_{\infty} \leq \gamma \Leftrightarrow |H(j\omega)| \leq \gamma, \forall \omega \in [0, \infty)$$

Sensitivity functions (gang of four)

$$S = \frac{1}{1 + C'P}, T = C'PS, S_C = C'S, S_P = PS$$

Weighting functions

$$W_s(s), W_T(s), W_C(s), W_P(s),$$

Servo problem

(set-point tracking)

$$r \rightarrow e' : \|W_s S\|_{\infty} \leq M_s$$

$$r \rightarrow y' : \|W_T T\|_{\infty} \leq M_T$$

$$n \rightarrow u' : \|W_C S_C\|_{\infty} \leq M_C$$

Regulator problem

(load disturbance rejection)

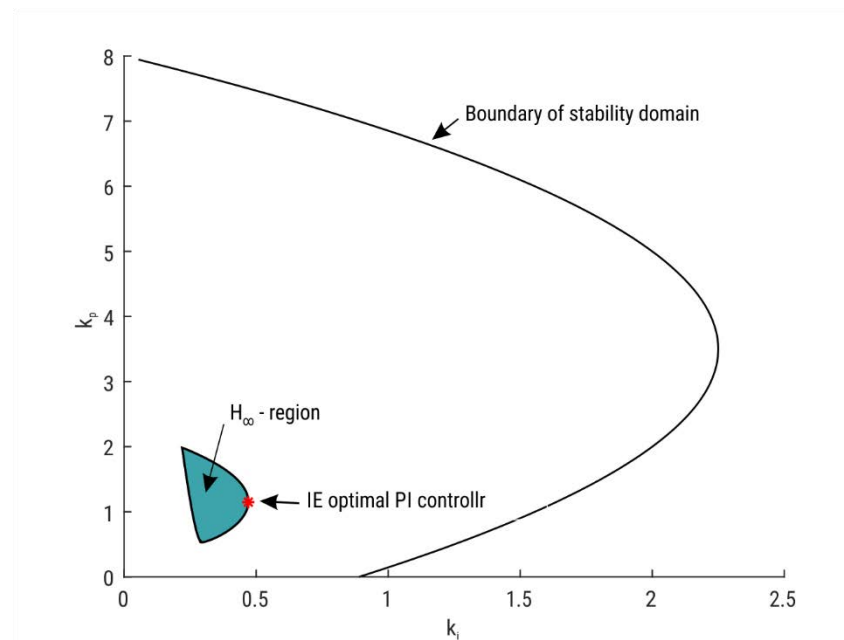
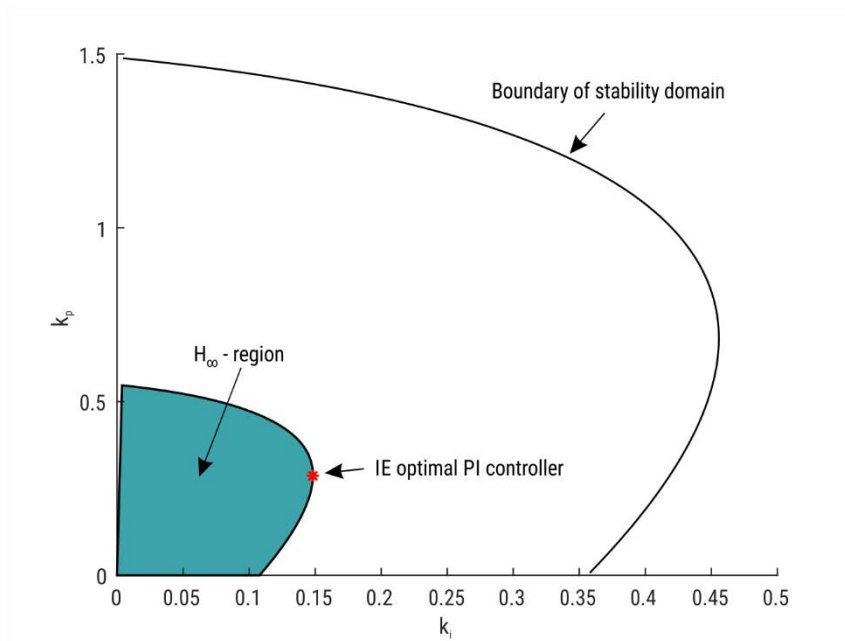
$$d_i \rightarrow y'' : \|W_P S_P\|_{\infty} \leq M_P$$

$$d_o \rightarrow e' : \|W_s S\|_{\infty} \leq M_T$$

$$n \rightarrow u' : \|W_C S_C\|_{\infty} \leq M_C$$

H_∞ -Region in the Parametric Plane $k_i - k_p$

(It contains all PI controllers that meet the specified H_∞ limitations)



Finding the H_∞ - region

$$\mathbf{K} \triangleq \left\{ k = \begin{bmatrix} k_i \\ k_p \end{bmatrix} : \|H(s, k)\|_\infty \leq \gamma, \text{ the closed-loop is stable} \right\}$$

is generally a very difficult problem. PID Hinf Designer (www.pidlab.com) is the first software tool available to fully address this issue.

Example of Simple Design specification of PI-controller for FOPDT system

Proces transfer function:	$P(s) = \frac{e^{-s}}{s + 1}$
Controller transfer function:	$C_{PI}(s) = K \left(1 + \frac{1}{T_i s} \right) = k_p + \frac{k_i}{s}$
Sensitivity function:	$S(s) = \frac{1}{1 + C_{PI}(s)P(s)}$
Weighting function:	$W_s(s) = 1$
Type of control problem:	regulator problem (load step disturbance rejection)

Design specification:	$IAE = \min_{C_{PI}} \int_0^{\infty} e(t) dt$ <p>subject to $\ S(s)\ _{\infty} \leq M_s \Leftrightarrow S(j\omega) \leq M_s, \forall \omega \in [0, \infty)$</p>
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PID H_∞ Designer

Input :

$$P(s) = \frac{e^{-s}}{s+1}$$

$$\text{PI-controller} \left(C_{PI}(s) = k_p + \frac{k_i}{s} \right)$$

$$M_s = 1.6$$

Output :

$$IE : \quad k_p = 0.463, \quad k_i = 0.509$$

$$IAE : \quad k_p = 0.565, \quad k_i = 0.488$$

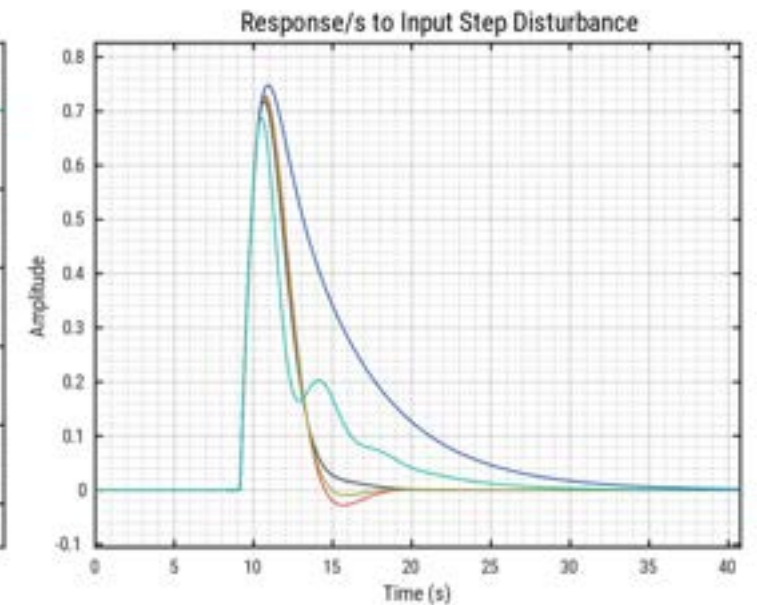
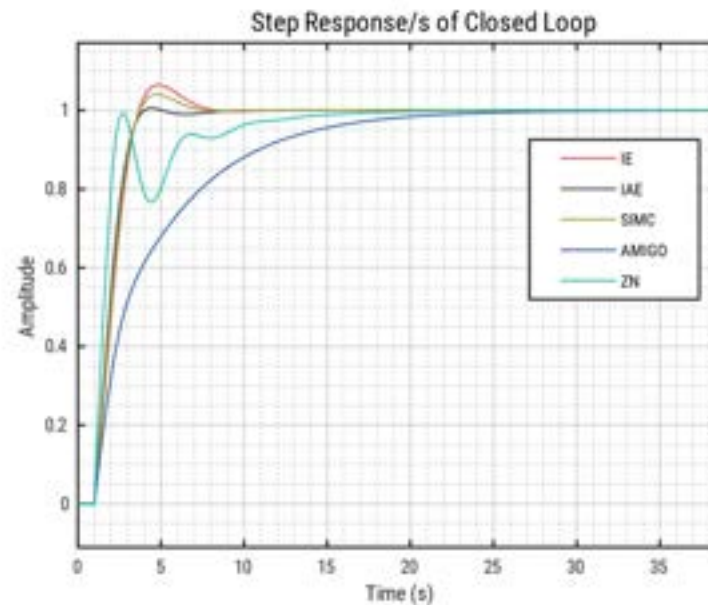
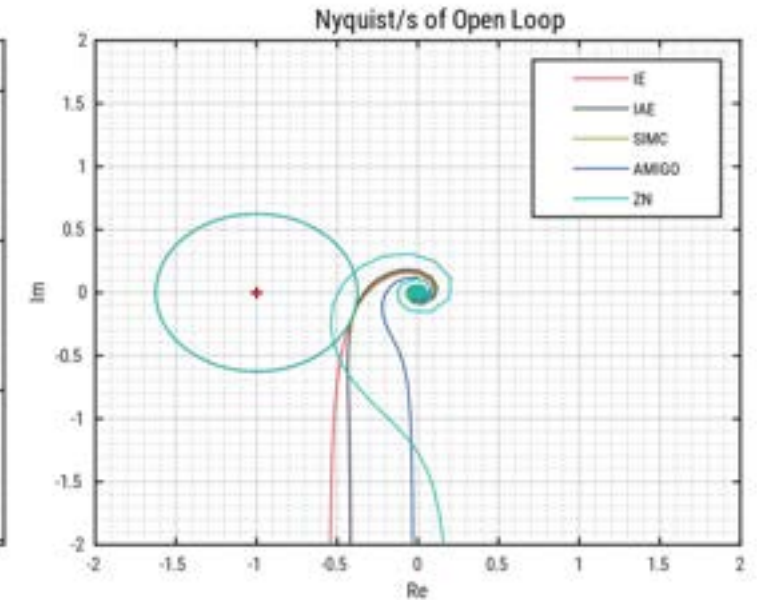
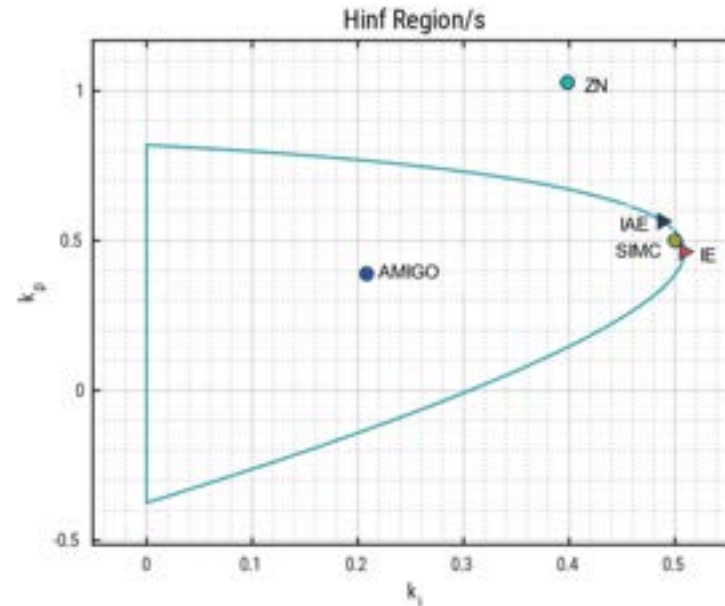
$$ITAE : \quad k_p = 0.557, \quad k_i = 0.492$$

Optional output :

ZN Ziegler-Nicols (1942, step response)

SIMC Skogestad (2012)

AMIGO Hagglund and Astrom (2004)





More General Formulation of Design Problem

(fully supported by PID H_∞ Designer)

$$\mathbf{P} = \{P_1, P_2, \dots, P_k\}$$

model set of transfer functions

$$P \in \mathbf{P}$$

process transfer function

$$C \in \{C_{PI}, C_{PID}\}$$

controller transfer function

$$S = \frac{1}{1 + CP}, \quad T = \frac{CP}{1 + CP}, \quad S_C = \frac{CP}{1 + CP}, \quad S_P = \frac{P}{1 + CP}$$

loop sensitivity transfer functions

$$\mathbf{I} = \{IE, IAE, ITAE, ISE\}$$

design criterion set

$$IE \triangleq \int_0^\infty e(t)dt, \quad IAE \triangleq \int_0^\infty |e(t)|dt, \quad ITAE \triangleq \int_0^\infty t|e(t)|dt, \quad ISE \triangleq \int_0^\infty e^2(t)dt$$

$$I \in \mathbf{I}$$

design criterion selected

$$W_S, W_T, W_C, W_P$$

weighting functions

Controller Robust Design Problem

$$\min_C \max_{P \in \mathbf{P}} I$$

subject to the H_∞ limitations

$$\forall P \in \mathbf{P}: \quad \|W_S S\|_\infty \leq M_S, \quad \|W_T T\|_\infty \leq M_S, \quad \|W_C S_C\|_\infty \leq M_C, \quad \|W_P S_P\|_\infty \leq M_P.$$

Example of Design Specification of Robust PI-controller for Process Model Set

Proces model set:	$\mathbf{P} \triangleq \left\{ P_1(s) = \frac{-0.0216s + 0.0031}{s^2 + 0.457s + 0.0868} e^{-0.166s}, P_2(s) = \frac{-0.0174s + 0.0046}{s^2 + 0.5978s + 0.0445} e^{-0.166s} \right\}$
Controller transfer function:	$C_{PI}(s) = K \left(1 + \frac{1}{T_i s} \right) = k_p + \frac{k_i}{s}$
Sensitivity functions:	$S_i(s) = \frac{1}{1 + C_{PI}(s)P_i(s)}, \quad i = 1, 2$
Weighting functions:	$W_i(s) = 1, \quad i = 1, 2$
Type of control problem:	regulator problem (load step disturbance rejection)

Design specification:	$\min_{C_{PI}} \max_{i \in \{1, 2\}} \int_0^\infty e_i(t) dt$ $\text{subject to } \ S_i(s)\ _\infty \leq M_s, \quad i = 1, \dots, 2 \Leftrightarrow S_i(j\omega) \leq M_s, \quad i = 1, 2, \quad \forall \omega \in [0, \infty)$
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PID H_∞ Designer

Input :

$$P_1(s) = \frac{-0.0216s + 0.031}{s^2 + 0.457s + 0.0868} e^{-0.166s}$$

$$P_2(s) = \frac{-0.0174s + 0.0445}{s^2 + 0.5978s + 0.0445} e^{-0.166s}$$

$$C_{PI}(s) = k_p + \frac{k_i}{s}$$

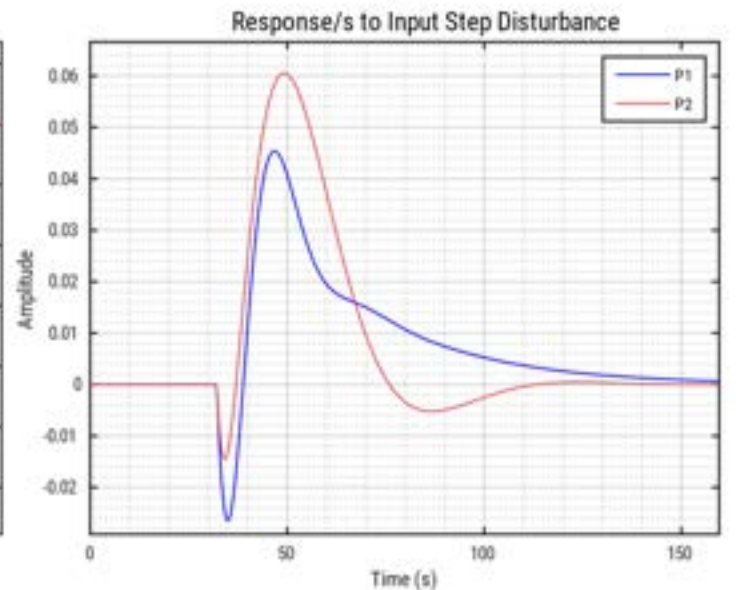
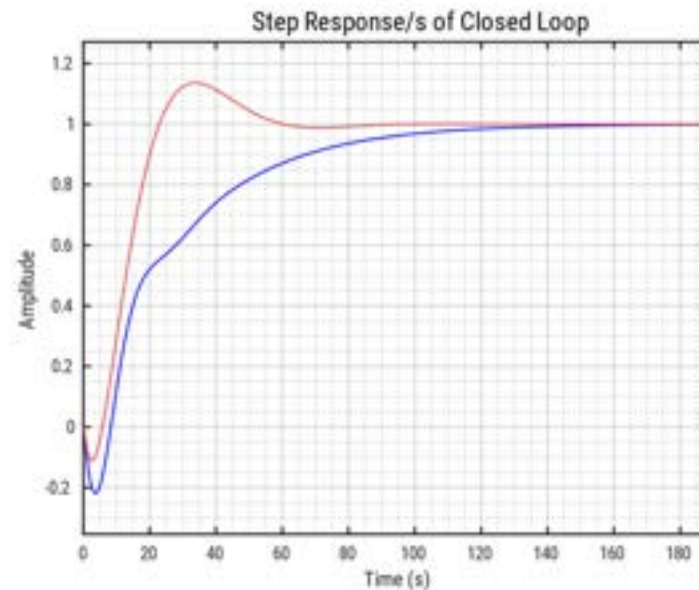
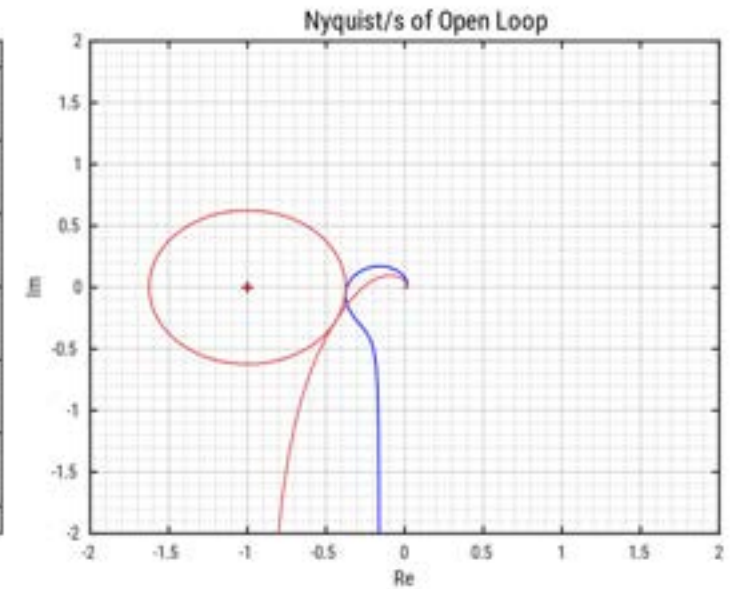
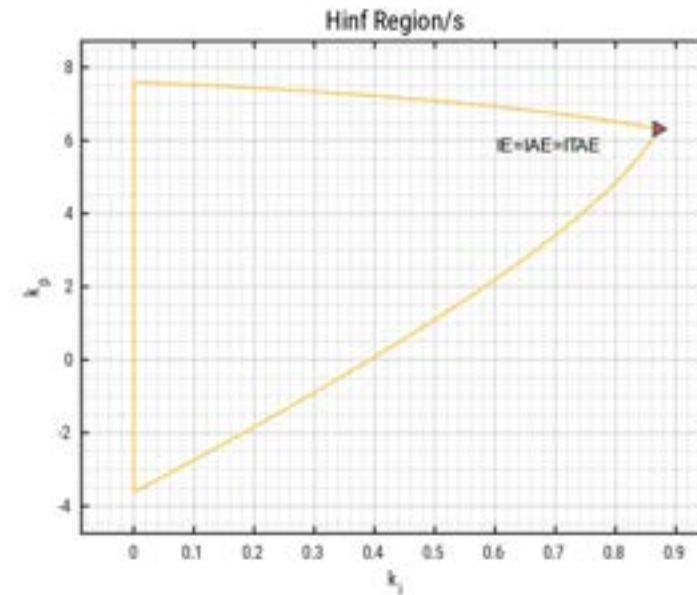
$$M_S = 1.6$$

Output :

$$IE : \quad k_p = 6.313, \quad k_i = 0.8704$$

$$IAE : \quad k_p = 6.313, \quad k_i = 0.8704$$

$$ITAE : \quad k_p = 6.313, \quad k_i = 0.8704$$



Conclusion (1)

- **PID H_∞ Designer** is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE and H_∞ robustness constraints.
- **PID H_∞ Designer** can be used for a wide range of process models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, ...) and also for so-called model sets created from any number of process transfer functions.
- **PID H_∞ Designer** provide a new explicit algorithm to determine the H_∞ - regions in the parameter plane of PI controller for all commonly used H_∞ limitations of the weighted sensitivity functions.
- **PID H_∞ Designer** also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one frequency point). Moreover, the non-standard moment model set provided by the **PIDMA-autotuner** from the company **REX Controls** is also supported.

Conclusion (2)

- Designing of PI(D) controller with typical specifications using **PID H_∞ Designer** is a routine procedure that does not require deeper knowledge of control theory from the user.
- With more skills and efforts from the designer it should be possible to design high performance PID controllers extended with any linear compensator suitable (Resonant Controller, Smith predictor, Repetitive Control, ...).
- More details about the affine-structured controller in Semi-Plenary Lecture or in white paper „*Analytical Design of a Wide Class of Controllers with Two Tunable Parameters Based on H_∞ Specifications*“

Semi-Plenary Lecture
Process Control 2023

H_∞ Affine Controller
White Paper

Appendix A: Isolation of H_∞ -Region (1)

For more details see: Schlegel M., Medvecová P., *Design of PI Controllers : H_{inf} Region Approach*. IFAC PapersOnLine 51-6 (2018), 13-17.

Proposition : If $C(s, k) = k_p + \frac{k_i}{s}$, $k = [k_i, k_p]$, $P(s)$ has no poles on the imaginary axis, and the design specification is

$$\|S(s, k)\|_\infty = \left\| \frac{1}{1 + C(s, k)P(s)} \right\|_\infty = \left\| \frac{S_n(s, k)}{S_d(s, k)} \right\|_\infty \leq \gamma \triangleq M_S \neq 1,$$

then the boundary of the H_∞ -region \mathbf{K} is contained in the solutions of the two systems of equations

$$\begin{aligned} \text{(i)} \quad & S_n(j\omega, k) = 0, & \text{(ii)} \quad & |S(j\omega, k)|^2 = \gamma^2, \\ & S_d(j\omega, k) = 0, & & \frac{\partial |S(j\omega, k)|^2}{\partial \omega} = 0. \end{aligned}$$

The system of equations (i) has a solution $k_i = 0$, i.e. any point on the axis k_p is a solution of this system. The solution of the system (ii) is determined by the parametric curves

Appendix A (2)

$$\left. \begin{aligned} k_i &= \frac{x_i \omega}{M_s}, \\ k_p &= \frac{\omega^4 + 2\omega^2 + 1}{\omega M_s^2 (2ABB_1 + A^2 A_1 - A_1 B^2)}, \end{aligned} \right\} \omega \in [0, \infty),$$

where A, A_1, B, B_1 are the functions of ω defined by

$$P(j\omega) = A(\omega) + jB(\omega) \triangleq A + jB,$$

$$\frac{dP(j\omega)}{d\omega} = A_1(\omega) + jB_1(\omega) \triangleq A_1 + jB_1,$$

and $x_i, i = 1, \dots, l(\omega)$, $l(\omega) \in \{0, 2, 4\}$ are the frequency dependent real roots of quartic polynomial

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

with the real frequency dependent coefficients

$$a = (A^2 + B^2)^4,$$

$$\begin{aligned} b &= 2M_s(A^2 + B^2)^2(-\omega B_1 A^2 + 2\omega B A A_1 + \omega B^2 B_1 + B A^2 + B^3), \\ &= -(\omega^4 + \omega^2)(2\omega^3 + 4\omega^2 + 2\omega + 1) - \omega^2(2\omega^2 + 2\omega + 1) - \omega^2(2\omega^2 + 2\omega + 1) - 8\omega^2 + 2\omega A A_1 B^2 - \omega^2 A_1^2 B^2 M_s^2 - B^4 M_s^2 + 2\omega B^3 B_1 - \omega^2 B^2 B_1^2 M_s^2 - 4\omega B^3 B_1 M_s^2), \end{aligned}$$

$$\begin{aligned} d &= -2\omega M_s(-\omega A_1^2 B^3 M_s^2 - \omega A^2 A_1^2 B M_s^2 - 2A A_1 B^3 M_s^2 - \omega A^2 B B_1^2 M_s^2 + A^2 B^2 B_1 M_s^2 - B^4 B_1 M_s^2 - \omega B^3 B_1^2 M_s^2 + \\ &\quad + 3\omega^2 + 3\omega + 1 - \omega^2 + \omega^2 + \omega^3 + 2\omega^2 + \omega^3 + \omega^3 + \omega^2 + \omega^3 + \omega^4), \end{aligned}$$

$$e = -\omega^2(M_s^2 - 1)(-A_1^2 B^2 M_s^2 - B^2 B_1^2 M_s^2 + 2A A_1 B B_1 + B^2 B_1^2 + A^2 A_1^2).$$

Appendix A (3)

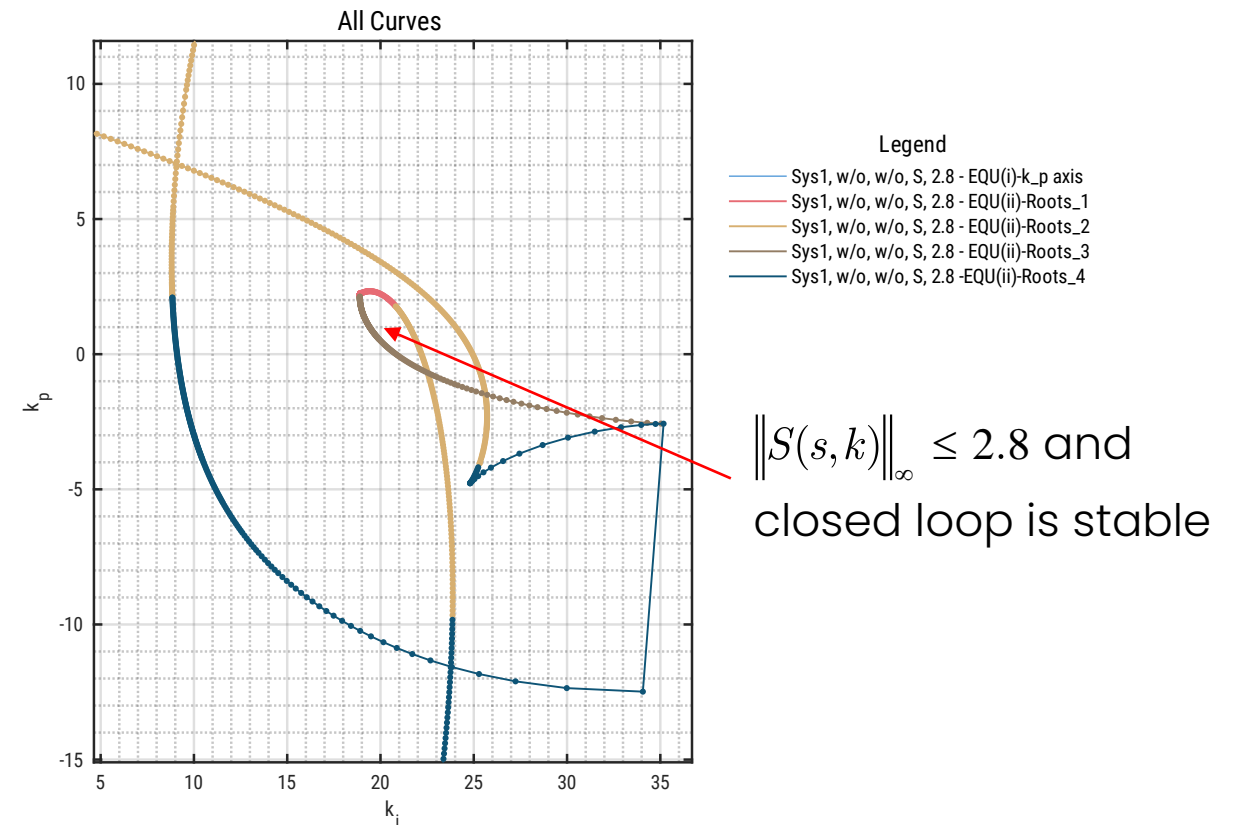
The curves representing the solutions of systems (i) and (ii) divide the parametric plane into regions. From them, it is necessary to select those that meet the design specifications. For this purpose, it is sufficient to test only one point of the respective region.

Example: H_∞ – region for unstable process:

$$P(s) = \frac{s^3 + 4s^2 - s + 1}{s^5 + 2s^4 + 32s^3 + 14s^2 - 4s + 50},$$

$$C(s, k) = k_p + \frac{k_i}{s}$$

$$\|S(s, k)\|_\infty = \left\| \frac{1}{1 + C(s, k)P(s)} \right\|_\infty \leq M_s = 2.8$$



Appendix A (4)

Workspace → Auxiliary Tools →
→ Multiparametric Analysis

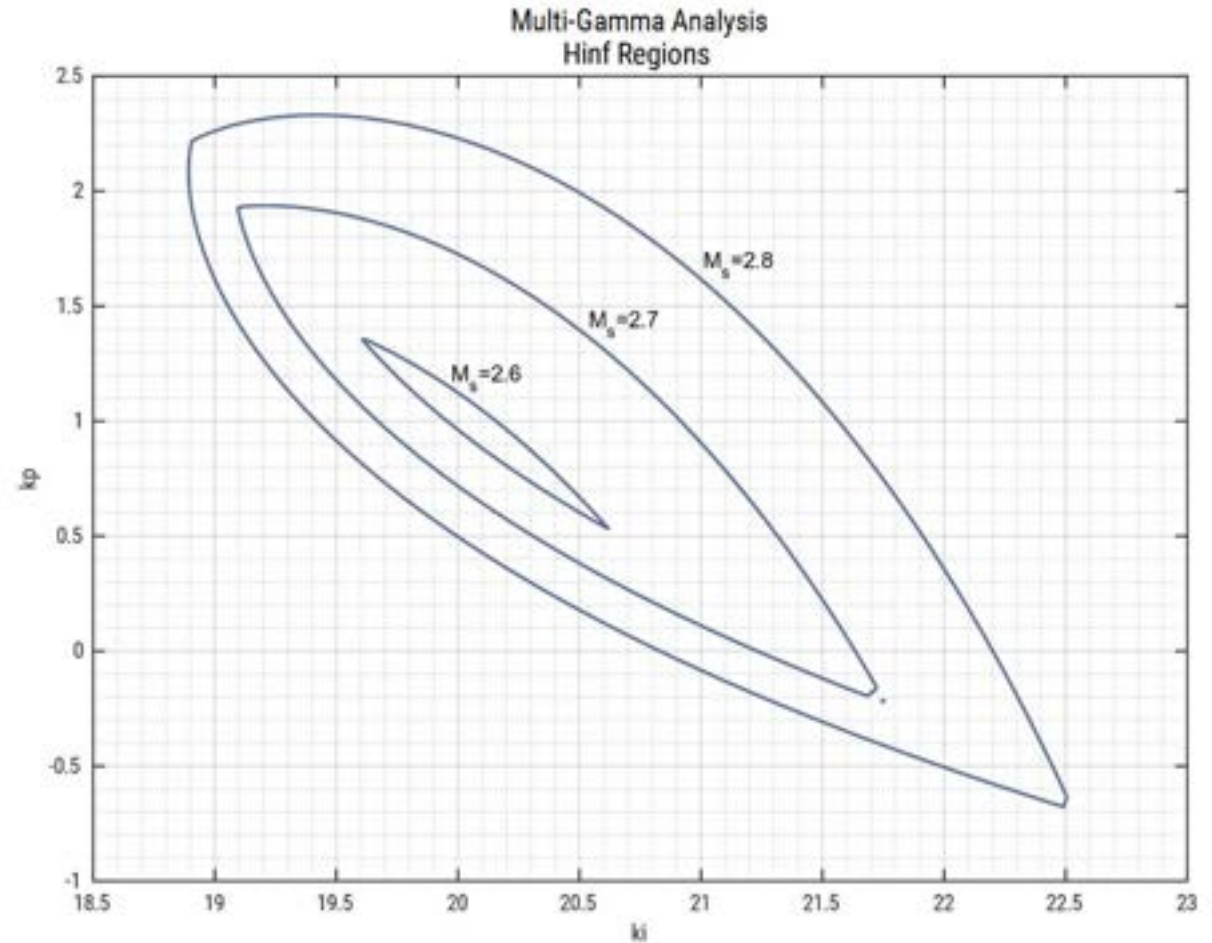
Example : H_∞ – region for unstable process:

$$P(s) = \frac{s^3 + 4s^2 - s + 1}{s^5 + 2s^4 + 32s^3 + 14s^2 - 4s + 50},$$


$$C(s, k) = k_p + \frac{k_i}{s}$$

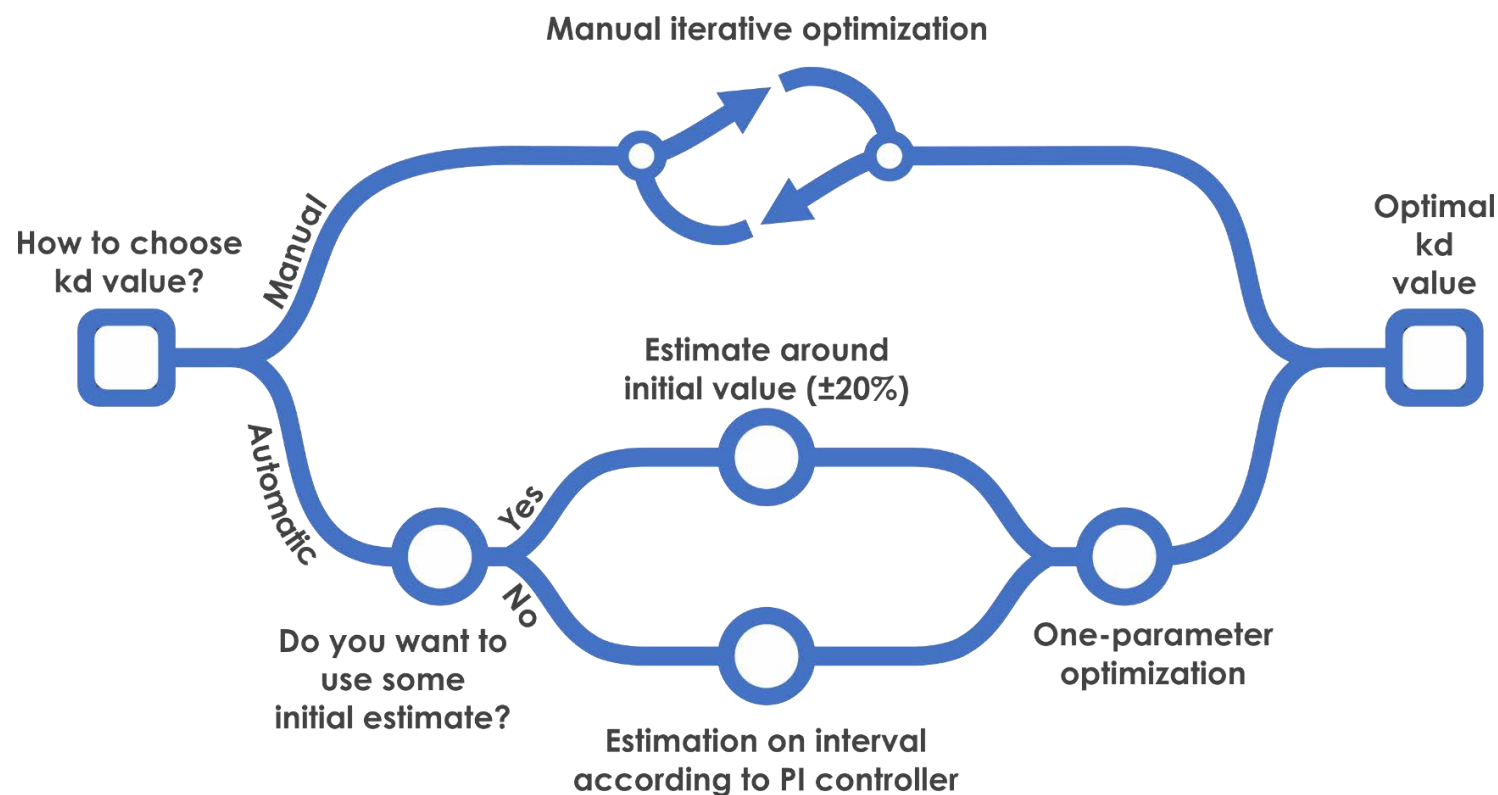
$$\|S(s, k)\|_\infty = \left\| \frac{1}{1 + C(s, k)P(s)} \right\|_\infty \leq M_s$$

$$M_s \in \{2.6, 2.7, 2.8\}$$



Appendix B: Selection of k_d and τ

It is recommended to start with the ideal PID controller ($\tau = 0$). If there exists a PI controller for the given design specification with parameters k_p, k_i , ($T_i = k_p / k_i$), then it is recommended to estimate optimal k_d in the interval $[0.2k_p^2 / k_i, 0.3k_p^2 / k_i]$ manually or with the help of GUI build-in function  .



Appendix C: PID H_∞ Designer GUI – Parametric Uncertainty (P.U.)



Specify the type of uncertainty and its parameters

Generating Outer Systems of a Model with Parametric Uncertainty

Specify the numerators and denominators of the transfer function of the controlled system with parametric value of a specific parameter is uncertain, replace it with the letter "u" or u(#) if there are more unknown parameters. The time delay of the controlled system can also be set as an uncertain parameter.

Num/Den Coeffs	Num/Den Str	Time Delay
Numerator <input type="text" value="u(1)"/>		Value <input type="text"/>
Denominator <input type="text" value="u(2)*s^2+u(3)*s+u(2)"/>		Order of Pade approximation (n-1)/n <input type="text"/>

e.g. No. 1: (u*s+1)*(u*s+1)^2*(s^2+1)
e.g. No. 2: (u(1)*s+1)*(u(2)*s+1)^2*(s^2+1)

< Back Next >

Define uncertain parameter/s in TF

Generating Outer Systems of a Model with Parametric Uncertainty

For each of the uncertain parameters of the transfer function of the controlled system, it is necessary to select its nominal value and variation.

$$P(s) = \frac{u_1}{u_2 s^2 + u_3 s + u_2}$$

Use to design the Smith Predictor: ☐

Variation Type:

Nominal value ± XX %. Uncertainty example: 10.

Parameters	Nominal Values	Uncertainty
u1	1	10
u2	1.5	5
u3	2.6	5

< Back Generate Cancel

Appendix D: PID H_∞ Designer GUI – Experimental Model Set (E.M.S.)



Select type of model set

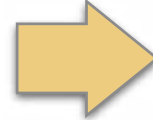
Select Approach

One point of the frequency response model set

Static gain and one point of the frequency response model set

PIDMA Moment model set

Cancel



Maximal order of system

Generating a Value Set Boundary of a Model Set

Generate systems of the model set described by the parameters κ , μ and σ^2

κ 1

μ 1

σ^2 0.6

n 5

Property Static

10

ω [rad/s] 5

Show Region

Generate Cancel

Model set parameters

Number of systems on the curve of value set of model set

Plot the value set at frequency omega

Appendix E: PID H_∞ Designer GUI – System Identification (1)



Time samples

Input data samples

System output samples

A user must use the clipboard function which is integrated into the window manager to enter Input/Output data. This tool is accessible through the control panel on the left side of the screen.

I/O Process Data

It is necessary to obtain experimental data to determine the model of the investigated system. This data represents a sequence of time instants and input/output values. It is advisable to choose a unit step function as test inputs. Inserting individual data arrays into the appropriate tables is realized via a clipboard.

Example

Clear Tables

I/O point assignment

Start time: 0

Sampling period: 0.1

End time: 240

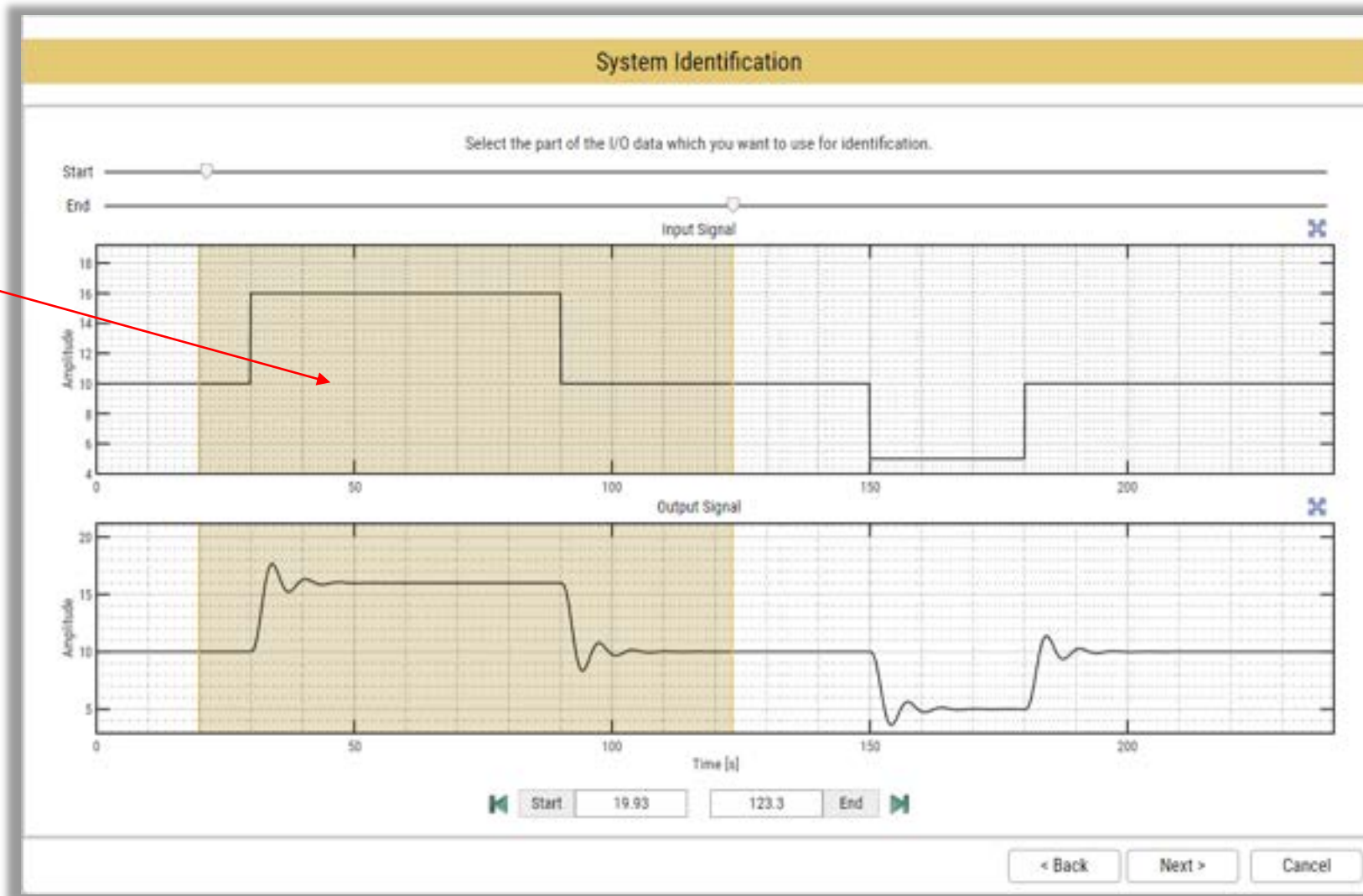
Time	Input	Output
1	0	10.0000
2	0.1000	10.0000
3	0.2000	10.0000
4	0.3000	10.0000
5	0.4000	10.0000
6	0.5000	10.0000
7	0.6000	10.0000
8	0.7000	10.0000
9	0.8000	10.0000
10	0.9000	10.0000
11	1.0000	10.0000
12	1.1000	10.0000
13	1.2000	10.0000
14	1.3000	10.0000
15	1.4000	10.0000
16	1.5000	10.0000
17	1.6000	10.0000
18	1.7000	10.0000
19	1.8000	10.0000
20	1.9000	10.0000
21	2.0000	10.0000
22	2.1000	10.0000
23	2.2000	10.0000
24	2.3000	10.0000
25	2.4000	10.0000

< Back Next > Cancel

Appendix E: PID H_∞ Designer GUI – System Identification (2)



Select part of
I/O data to
identification



Appendix E: PID H_∞ Designer GUI – System Identification (3)

The screenshot shows the 'System Identification' window of the PID H_∞ Designer. The interface includes a 'Model Design' section on the left, a central plot, and 'Actual Models' and 'Queue' sections on the right. Red arrows point to various elements with labels:

- Type of identified model:** Points to the 'Select the model type' dropdown menu, which is set to 'Oscillating Second Order Plus Dead Ti...'.
- Identified parameters of model:** Points to a red box containing the parameter fields for K (0.9969), ζ (0.364), ω (0.9815), and D (0.6302), along with a legend showing 'OSOPDT'.
- Run identification:** Points to the 'Estimate' button.
- Run additional numerical fitting:** Points to the 'Refine' button.
- Identified model types:** Points to the 'Actual Models' table.
- Delete selected actual model/s:** Points to the delete icon (trash can) in the 'Actual Models' section.
- Hide selected actual model/s response/s:** Points to the hide icon (eye with slash) in the 'Actual Models' section.
- Queue of models for importing to the System Editor:** Points to the 'Queue' table.
- Queue models editing:** Points to a red box around the icons for editing, deleting, and adding models in the 'Queue' section.
- Add selected/new model/s to the queue:** Points to the add icon (plus sign) in the 'Queue' section.

The central plot shows 'Amplitude' vs 'Time [s]'. It displays the 'Original' data (black line), the 'SOPDT' model (red line), the 'OSOPDT' model (green line), and the 'OSOPDT-model' (blue line). The plot shows a step response with oscillations.

The 'Actual Models' table lists the following models:

Type	RMSE	Show
SOPDT	0.3787	<input checked="" type="checkbox"/>
OSOPDT	0.0914	<input checked="" type="checkbox"/>

The 'Queue' table lists the following models:

Name	RMSE	Show	Upload
OSOPDT-model	0.0914	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>

The 'Transfer function' section displays the following equation:

$$P(s) = \frac{K\omega^2}{s^2 + 2\zeta\omega s + \omega^2} e^{-Ds}$$



Appendix F: Application Examples

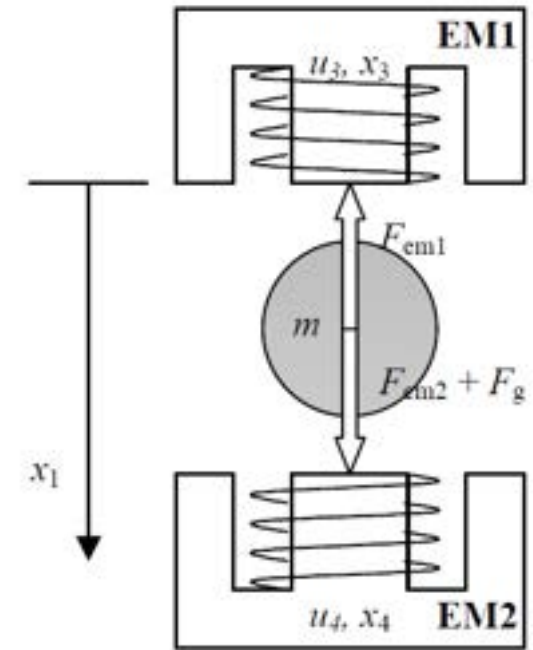
Magnetic Levitation System

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{F_{em1}}{m} + \frac{F_{em2}}{m} + g \\ \dot{x}_3 &= \frac{1}{f_i(x_1)}(k_i u_1 + c_i - x_3) \\ \dot{x}_4 &= \frac{1}{f_i(x_d - x_1)}(k_i u_2 + c_i - x_4)\end{aligned}$$

where

$$\begin{aligned}F_{em1} &= x_3^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_1}{F_{emP2}}} \\ F_{em2} &= x_4^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_d - x_1}{F_{emP2}}} \\ f_i(x) &= \frac{f_{iP1}}{f_{iP2}} e^{-\frac{x}{f_{iP2}}}\end{aligned}$$

F_{em1} —attraction force of the upper electromagnet [N],
 F_{em2} —attraction force of the lower electromagnet [N],
 F_g —force of gravity [N],
 g —acceleration of gravity—9.81 [m/s²]
 m —mass of ball—0.0571 [kg],
 u_1 —electric voltage of the upper coil— $\langle u_{min}, 1 \rangle$,
 $u_{min} = 0.00498$ [V],
 u_2 —electric voltage of the lower coil— $\langle u_{min}, 1 \rangle$ [V],
 x_d —distance between the magnets minus the ball diameter—defined by user [m],
 x_1 —distance from the upper magnet to ball
 $\langle 0, 0.016 \rangle$ [m],
 x_2 —linear speed of the ball [m/s]
 x_3 —coil current of the upper electromagnet
 $\langle i_{min}, 2.38 \rangle$,
 $i_{min} = 0.03884$ [A],
 x_4 —coil current of the lower electromagnet
 $\langle i_{min}, 2.38 \rangle$ [A].



$$c_i = 0.0242 \text{ [A]}$$

$$F_{emP1} = 1.7521 \times 10^{-2} \text{ [H]}$$

$$F_{emP2} = 5.8231 \times 10^{-2} \text{ [H]}$$

$$f_{iP1} = 1.4142 \times 10^{-4} \text{ [ms]}$$

$$f_{iP2} = 4.5626 \times 10^{-3} \text{ [m]}$$

$$k_i = 2.5165 \text{ [A]}$$

Magnetic Levitation System: Linear Model Set

Transfer Functions from u_1 to x_1 ($u_2=0$)

$$P_1(s) = \frac{-2.0893e4}{s^3 + 186.2891 \cdot s^2 - 1.6847e3 \cdot s - 3.1384e5}, \quad (x_1 = 8[\text{mm}])$$

$$P_2(s) = \frac{-2.7277e4}{s^3 + 288.7746 \cdot s^2 - 1.6847e3 \cdot s - 4.8649e5}, \quad (x_1 = 10[\text{mm}])$$

$$P_3(s) = \frac{-3.5611e4}{s^3 + 447.6417 \cdot s^2 - 1.6847e3 \cdot s - 7.5413e5}, \quad (x_1 = 12[\text{mm}])$$

[ML1] Hypiusová M., Kozáková A.: *Robust PID Controller Design for the Magnetic Levitation System: Frequency Domain Approach*. 21st International Conference on Process Control (PC), June 6-9, 2017, Štrbské Pleso, Slovakia

PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3\}$

Design specification:

2DOF PID controller

Setpoint tracking, IAE

$$M_S \leq 2.0, \quad M_T \leq 1.7$$

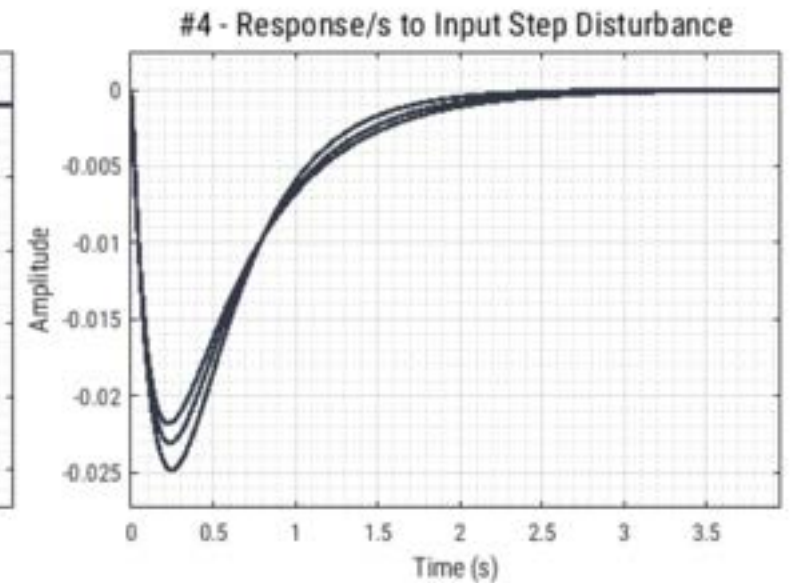
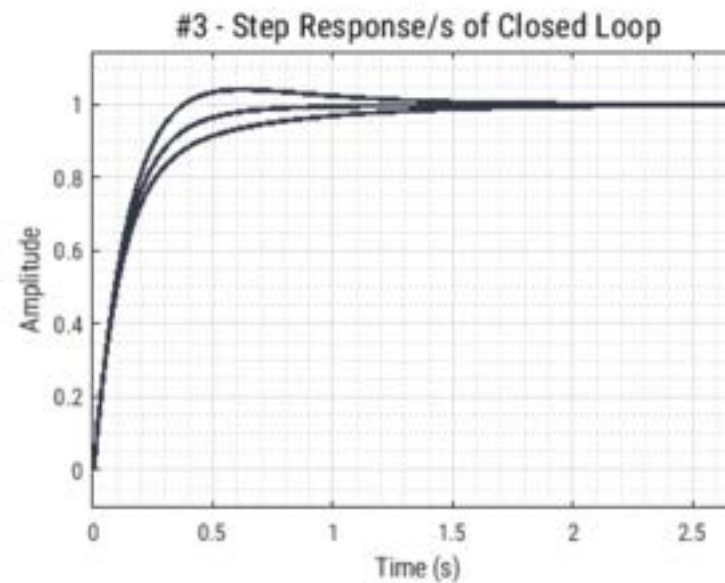
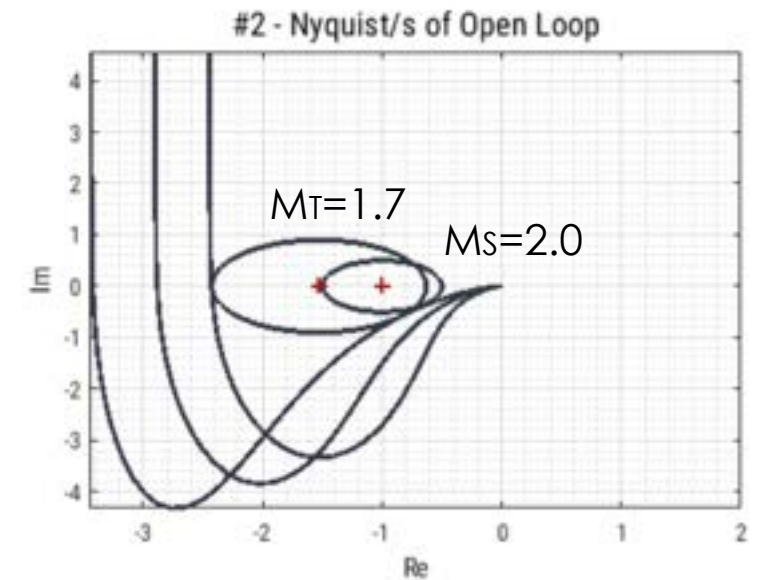
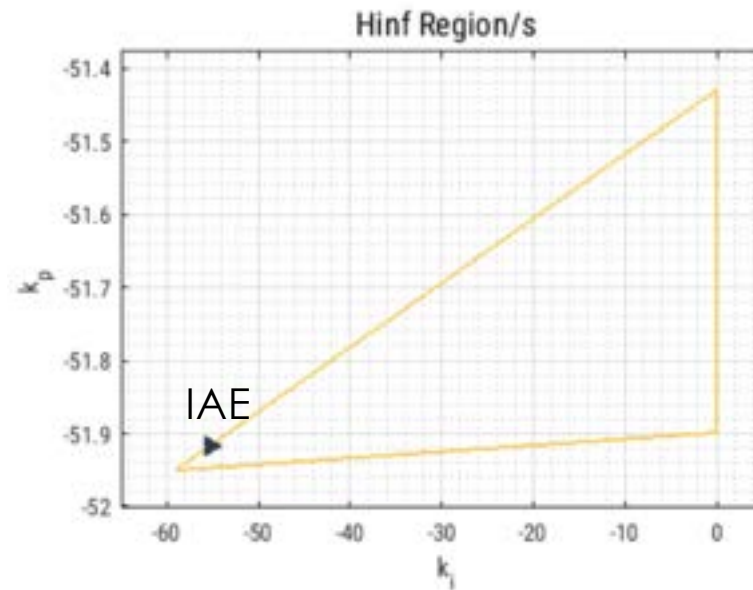
Output :

$$k_p = -51.95$$

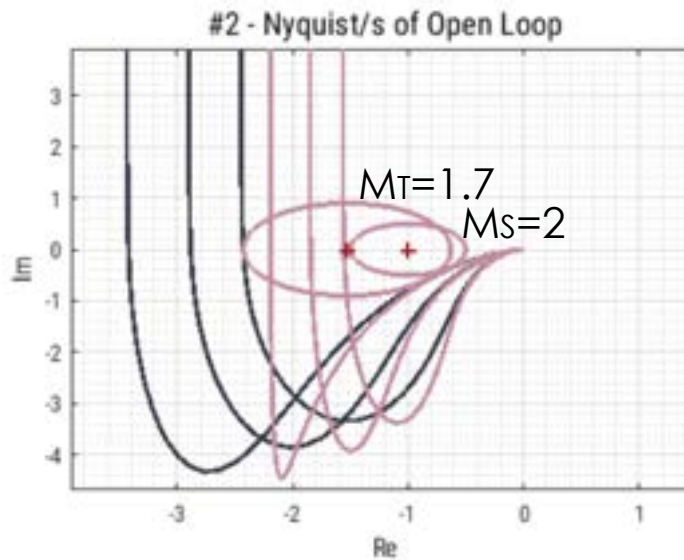
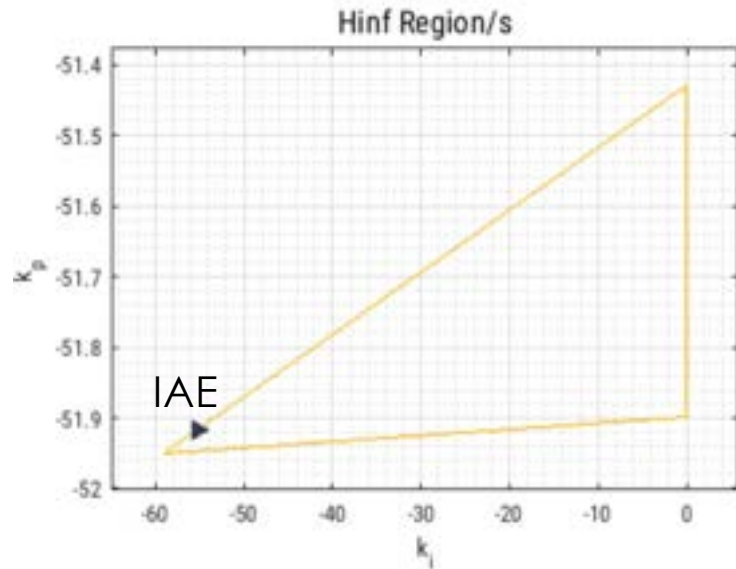
$$k_i = -59.07$$

$$k_d = -3.63$$

$$b = 0.5, \quad c = 0.0$$



Comparison with the PID-controller proposed in [ML1]



PID H_∞ Designer: —

2DOF PID - controller

$$k_p = -51.95$$

$$k_i = -59.07$$

$$k_d = -3.63$$

$$b = 0.5, c = 0.0$$

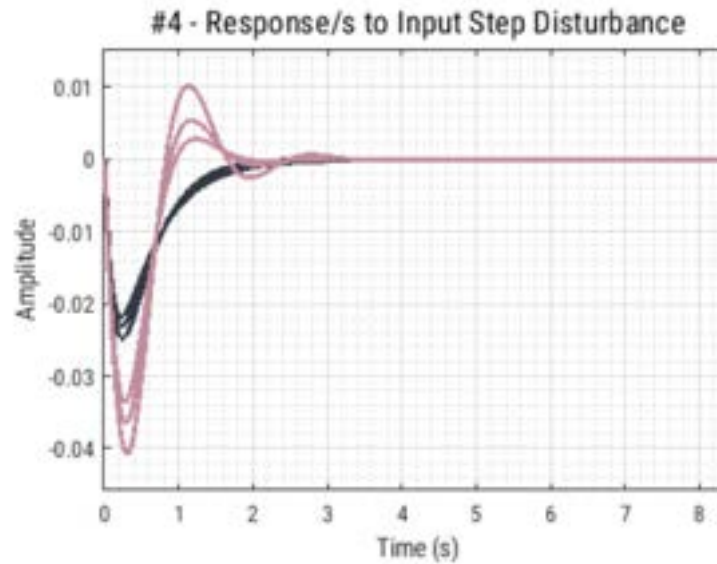
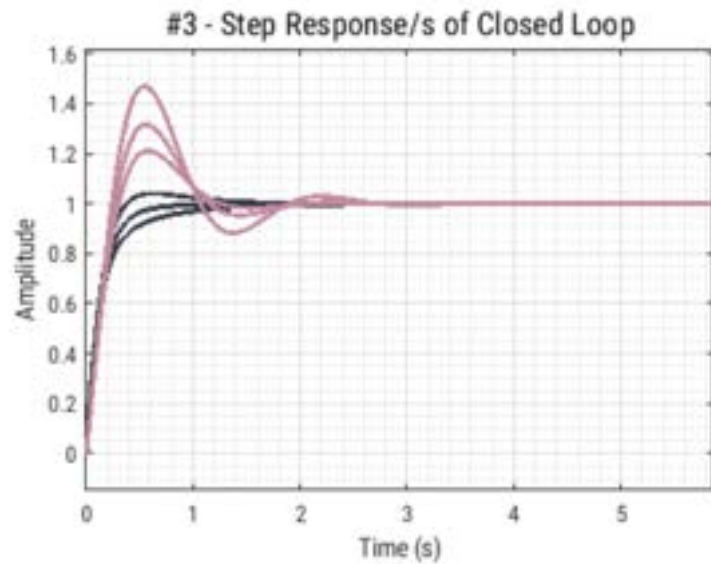
[ML1]: —

1DOF PID - controller

$$k_p = -33.27$$

$$k_i = -61.04$$

$$k_d = -4.532$$



Longitudinal motion of F4E fighter aircraft

We consider a model of the longitudinal motion of an F4E fighter aircraft [LM1], [LM2]. The input is the elevator position, the output is the pitch rate, and the system is linearized around four representative flight conditions:

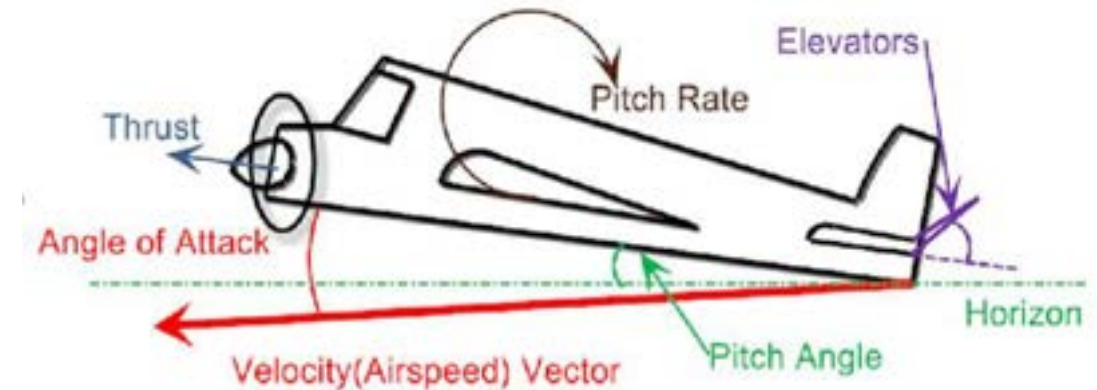
$$P_i(s) \triangleq \frac{b^i(s)}{a^i(s)}, \quad i = 1, \dots, 4.$$

Mach 0.5, 5000 ft: $a^1(s) = -52.75 + 22.00s + 15.84s^2 + s^3$, $b^1(s) = -163.8 - 185.4s$

Mach 0.85, 5000 ft: $a^2(s) = -122.5 + 34.93s + 17.12s^2 + s^3$, $b^2(s) = -789.1 - 507.8s$

Mach 0.9, 35000 ft: $a^3(s) = -14.64 + 17.51s + 15.33s^2 + s^3$, $b^3(s) = -101.8 - 158.3s$

Mach 1.5, 35000 ft: $a^4(s) = 269.1 + 43.60s + 15.74s^2 + s^3$, $b^4(s) = -251.4 - 304.2s$



[LM1] J. Ackermann. *Robust Control Systems with Uncertain Physical Parameters*. Springer Verlag, Berlin, 1993.

[LM2] Henrion D., Šebek M., Kučera V.: *Positive polynomials and robust stabilization with fixed - order controllers*. IEEE Trans. Automatic Control AC-48 (2003), 7.

PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PI controller

Setpoint tracking, IAE

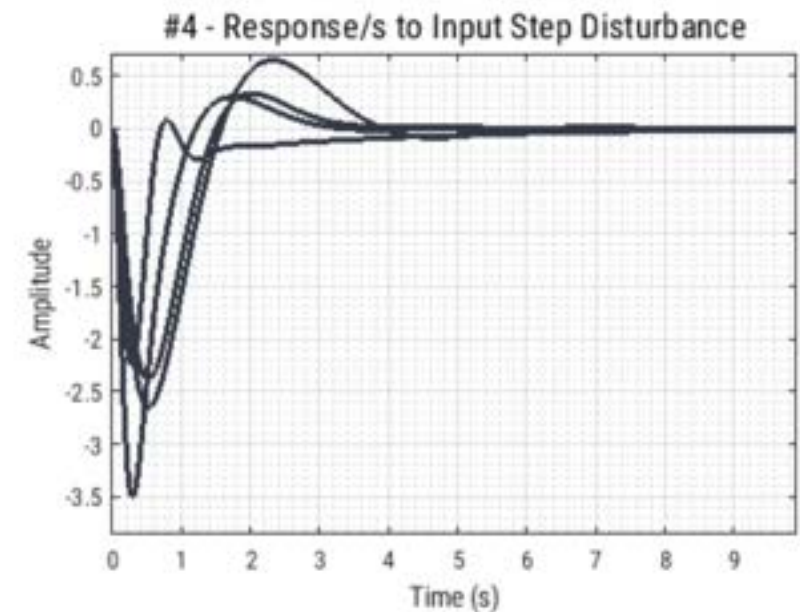
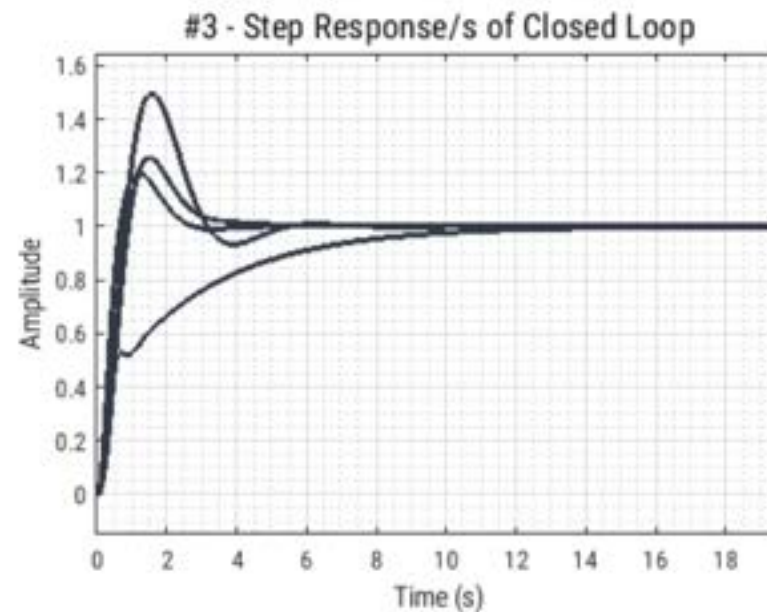
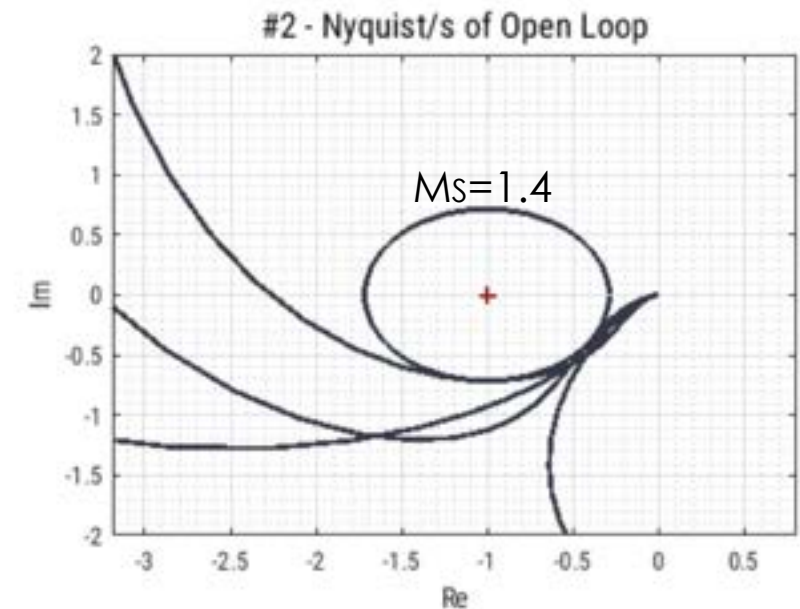
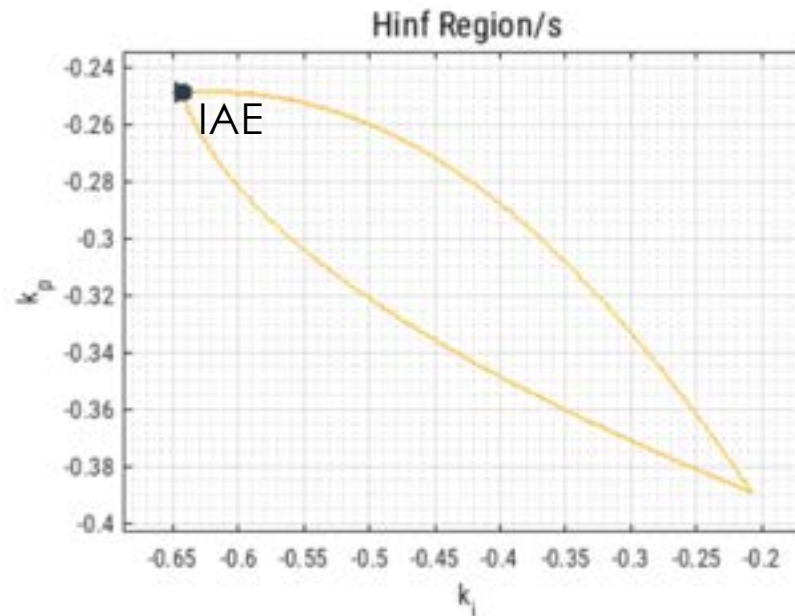
$$M_s \leq 1.4$$

Output :

$$k_p = -0.25$$

$$k_i = -0.64$$

$$b = 0.0, \quad c = 0.0$$



PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$$M_S \leq 1.4, \quad M_T \leq 1.4$$

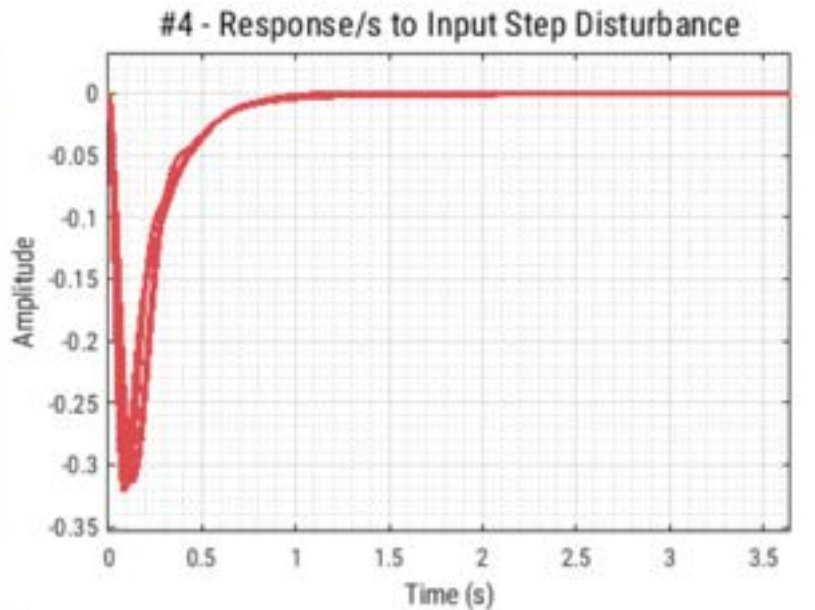
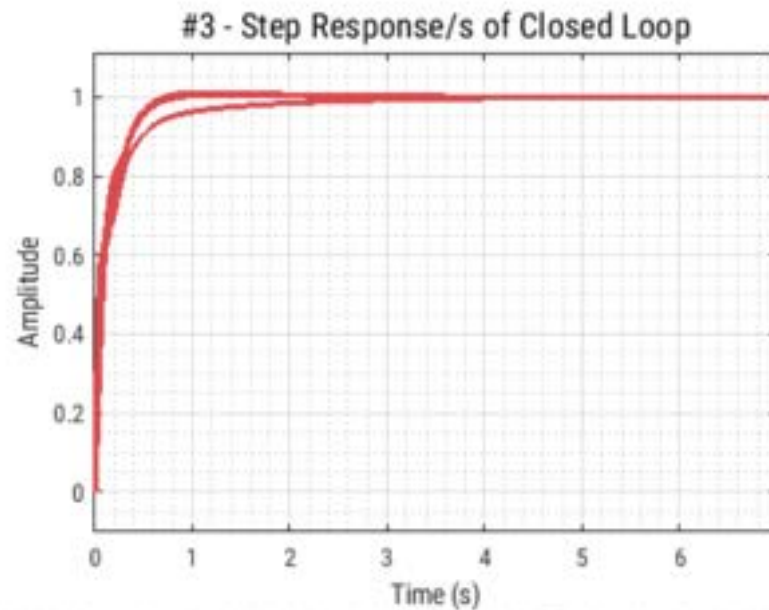
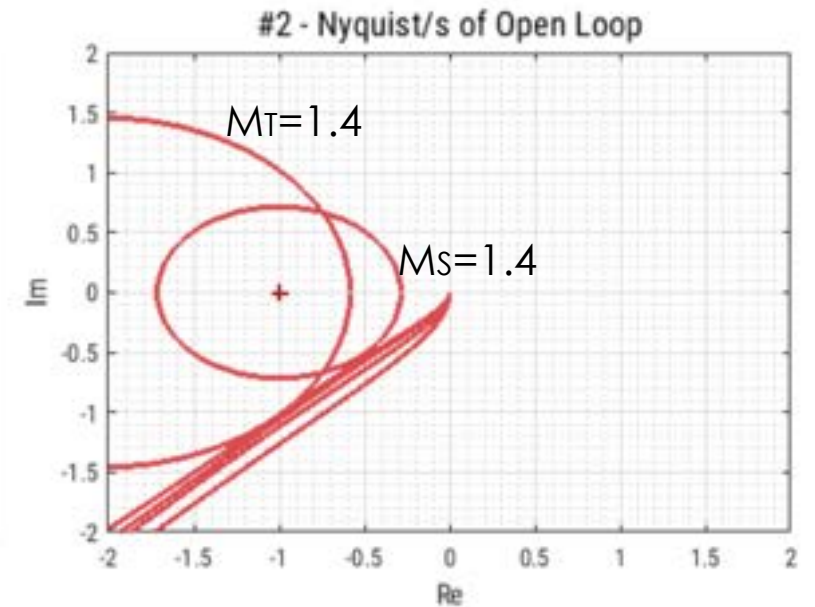
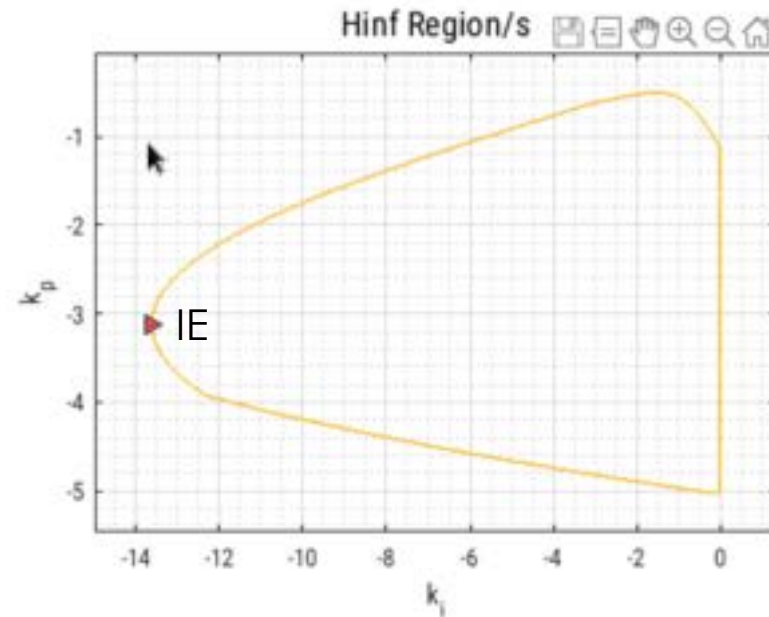
Output :

$$k_p = -3.12$$

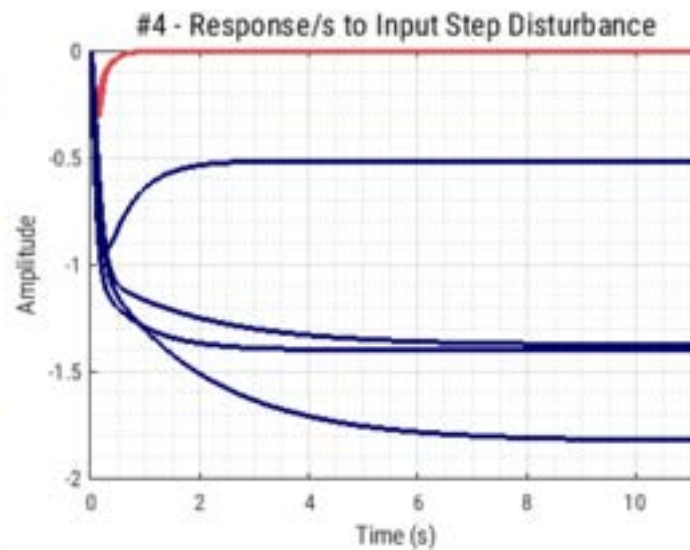
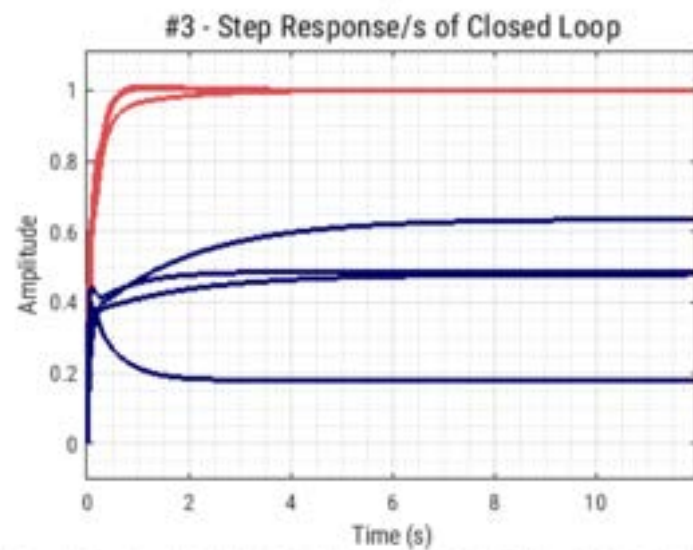
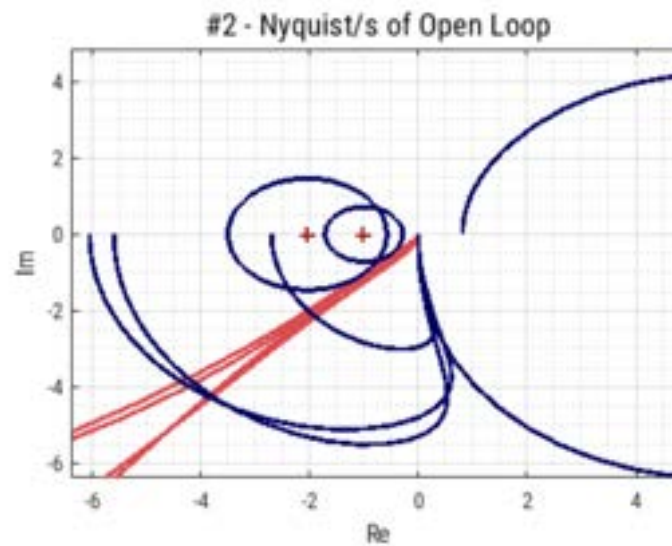
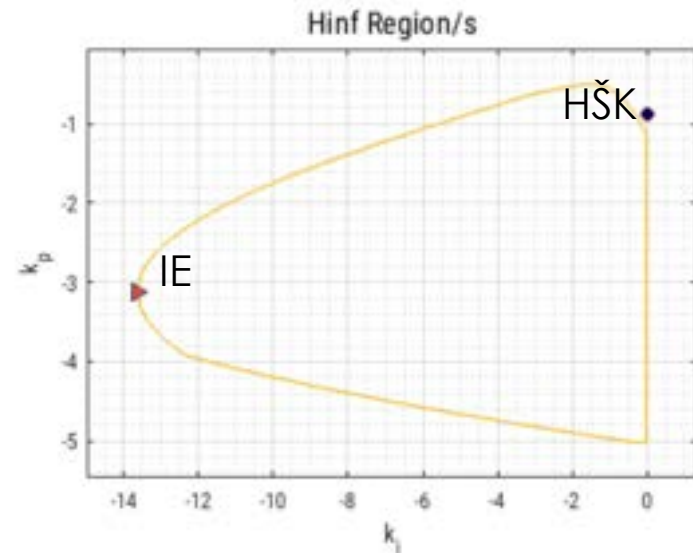
$$k_i = -13.63$$

$$k_d = -0.06$$

$$b = 0.4, \quad c = 0.6$$



Comparison with the P-controller proposed in [LM2]



PID H_∞ Designer: —

2DOF PID - controller

$$k_p = -3.12$$

$$k_i = -13.63$$

$$k_d = -0.06$$

$$b = 0.4, \quad c = 0.6$$

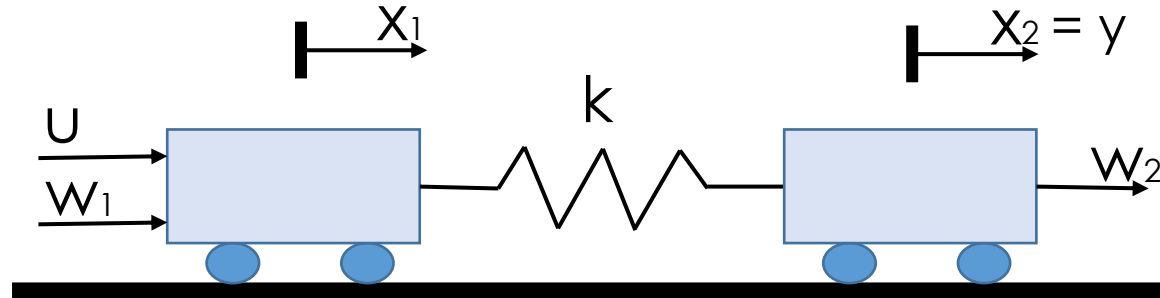
HŠK [LM2]: —

P - controller

$$k_p = -0.8698$$

Benchmark Problem for Robust Control

Wie, B. and D.S. Bernstein (1990). A benchmark problem for robust control design. In: *Proc. American Control Conference*. San Diego, CA, USA. pp. 961–962.



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k/m_1 & k/m_1 & 0 & 0 \\ k/m_2 & -k/m_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1/m_1 & 0 \\ 0 & 1/m_2 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

$$P_{ux_2}(s, k) = \left\{ \frac{k}{s^2(s^2 + 2k)} : k \in [0.5, 2] \right\} \rightarrow P(s, k, \xi) = \frac{k}{s(s^2 + 2\xi\sqrt{2k} \cdot s + 2k)}, \quad \xi \rightarrow 0$$

\uparrow $PD \ (^2k_p = {}^1k_i, {}^2k_d = {}^1k_p)$ \downarrow $PI \ ({}^1k_p, {}^1k_i)$

PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3\}$

$$P_1(s) = P(s, 0.5, 0.1),$$

$$P_2(s) = P(s, 1.0, 0.1),$$

$$P_3(s) = P(s, 2.0, 0.1).$$

Design specification:

1DOF PI + compensator $F(s)$

$$F(s) = \left(\frac{\Omega^2}{(s^2 + 2\xi\Omega s + \Omega^2)} \right)^2$$

$$\Omega = 0.9, \quad \xi = 0.7$$

Setpoint tracking, IE

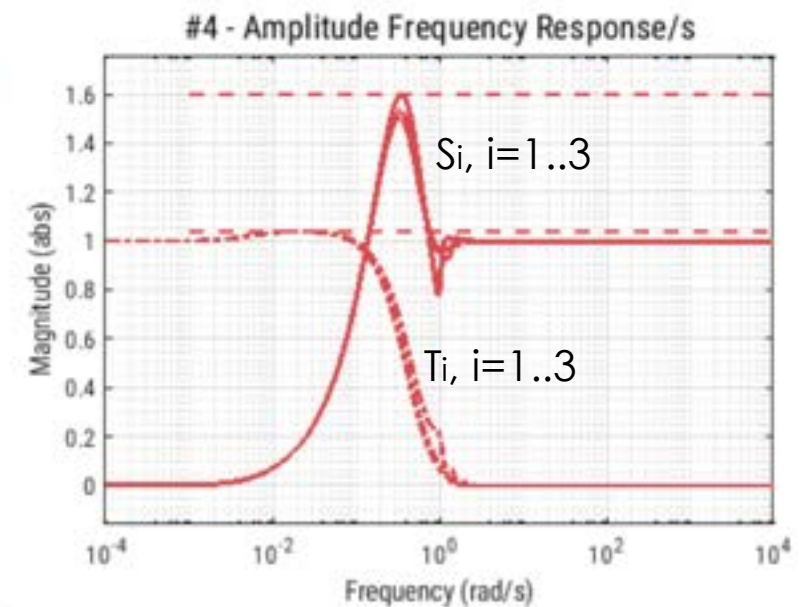
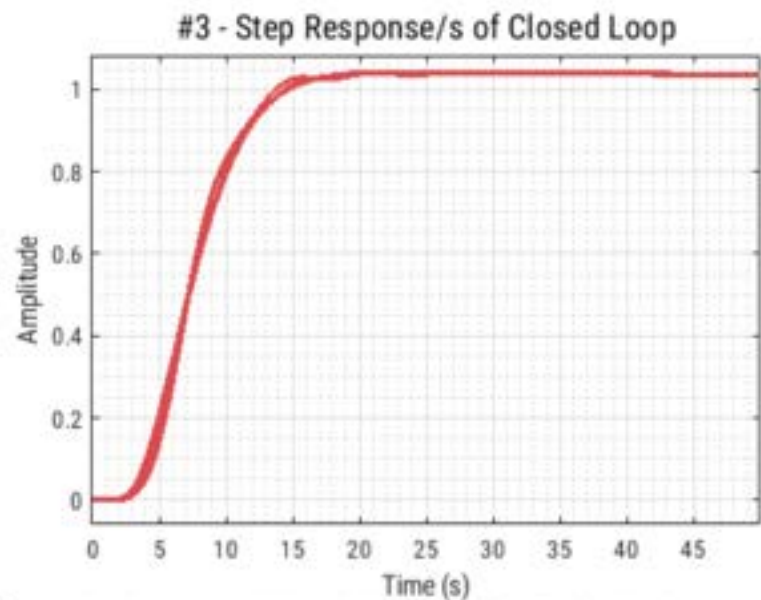
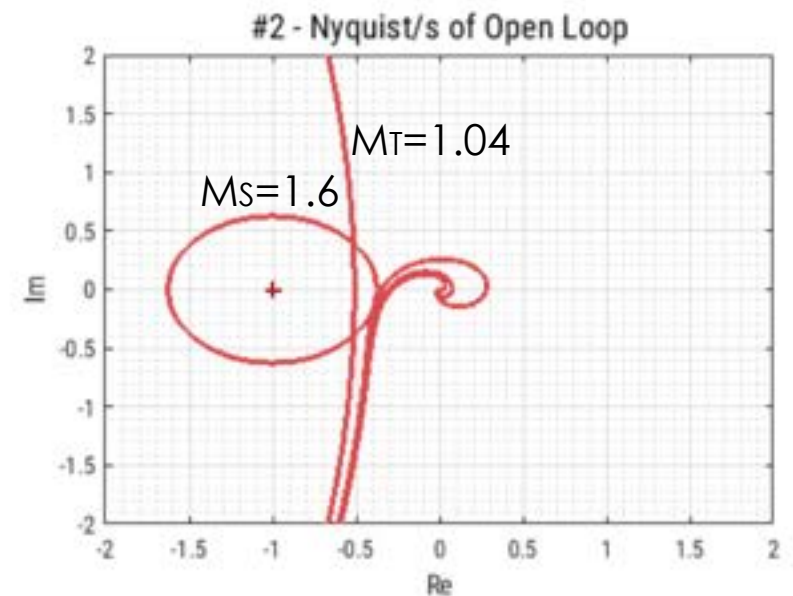
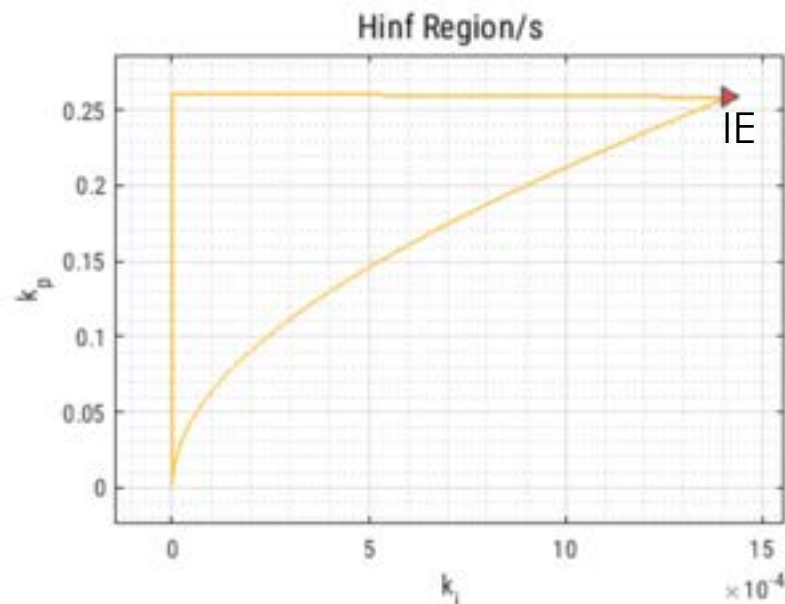
$$M_s \leq 1.4, \quad M_T \leq 1.05$$

Output :

$$k_p = 0.2586$$

$$k_i = 0.001413$$

$$b = 0.0$$



PID Controller Design using One Frequency Point



- SCHLEGEL, M.: *Nový přístup k robustnímu návrhu průmyslových regulátorů*. Habilitační práce, Západočeská univerzita v Plzni, 2000. <https://www.schlegel.zcu.cz/downloads.php?lng=eng>
- SCHLEGEL, M.: *Exact Revision of the Ziegler-Nichols Frequency Response Method*. In Proceedings of the IASTED International Conference Control and Application, Cancun, Mexico, 2002, p. 121-126. ISBN 088986330X, ISSN 1025-8973 .

Definition (One Point Model Set). We are given one disturbance free sample of the plant frequency response $G(j\omega_1)$ and a fixed $n \in \{2, \dots, \infty\}$. A plant model $P(s)$ is an element of the plant family $\mathcal{P}_n(G(j\omega_1))$ if it is consistent with the two following conditions:

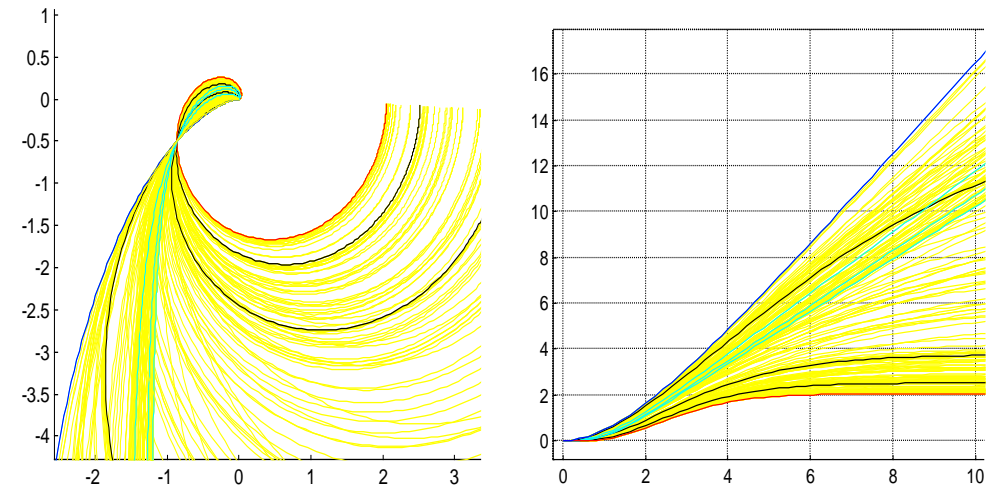
(i) (A priori Hypothesis)

$$P(s) = \frac{1}{p(s)},$$

where $p(s)$, $\deg(p(s)) \leq n$, is a polynomial with real nonnegative coefficients, and all roots of $p(s)$ lie in the interval $(-\infty, 0]$.

(ii) (Experimental Data Interpolation)

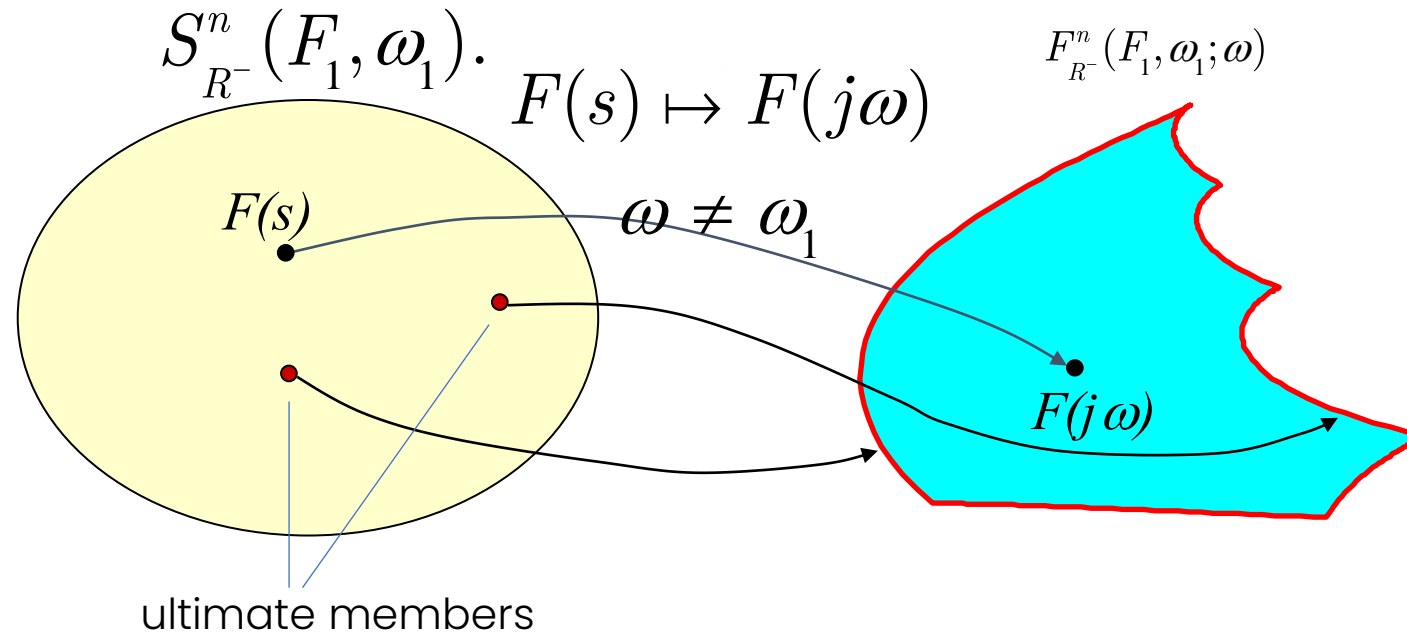
$$P(j\omega_1) = F_1, \quad -2\pi < \arg P(j\omega_1) \leq 0.$$



Main Idea of Solution

model set of dimension $n-2$

value set of dimension 2



Only ultimate members of the unfalsified plant family can play an active role in the Nyquist curve constraints.

PID H_∞ Designer

Input :

Model Set:

$$S_{\mathbb{R}^-}^n(F_1, \omega_1),$$

$$n = 10, F_1 = e^{-1.8j}, \omega_1 = 1$$

Design specification:

2DOF PI controller

Setpoint tracking, IE

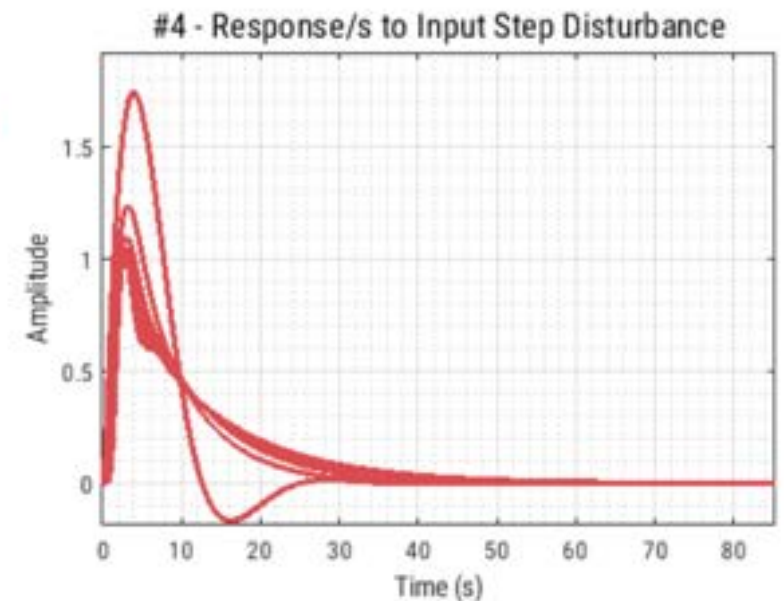
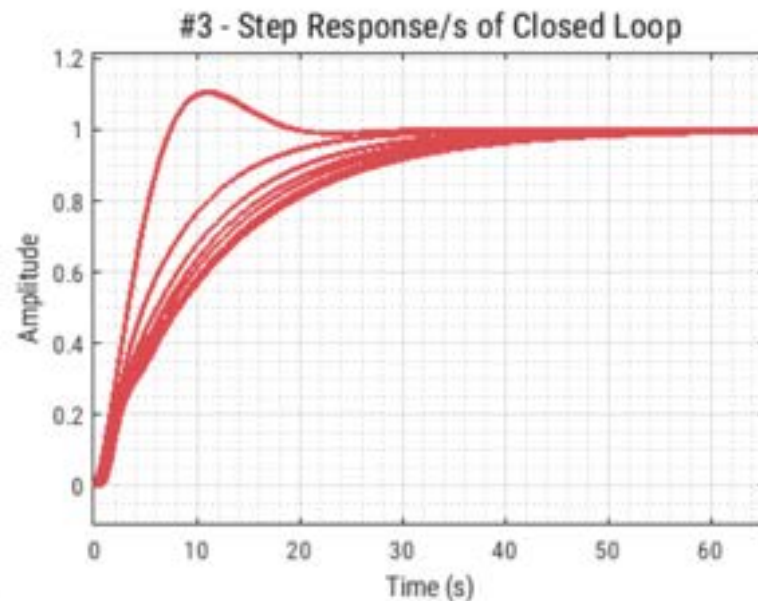
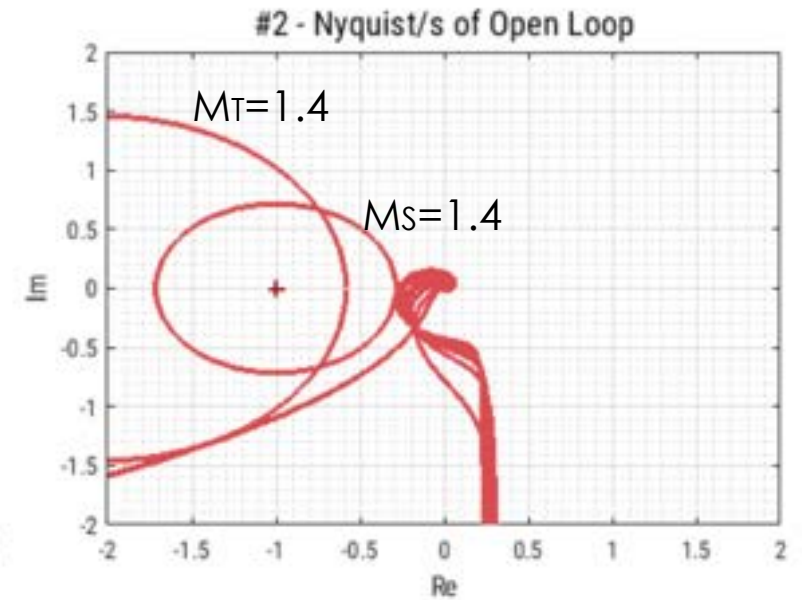
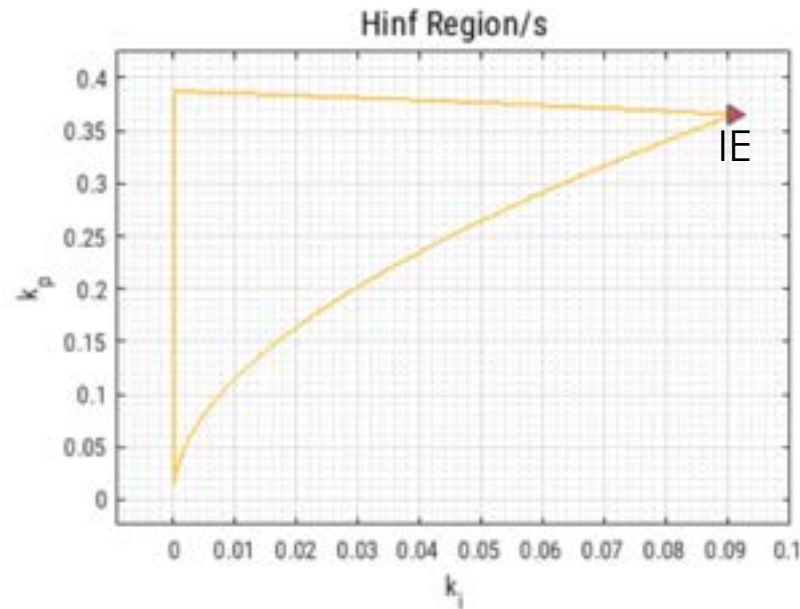
$$M_s \leq 1.4, M_T \leq 1.4$$

Output :

$$k_p = 0.37$$

$$k_i = 0.091$$

$$b = 0.3$$



- SCHLEGEL M.: *Nový přístup k robustnímu návrhu průmyslových regulátorů*. Habilitační práce, Západočeská univerzita v Plzni, 2000. <https://www.schlegel.zcu.cz/downloads.php?lng=eng>
- SCHLEGEL M., Večerek O.: *Robust design of Smith predictive controller for moment model set*. Proceedings of the 16th IFAC World Congress, p. 427-432, Elsevier, Oxford, 2006.
- SCHLEGEL M., BALDA P., ŠTĚTINA M.: *Robustní PID autotuner: momentová metoda*. Automatizace, 46(4):242–246, 2003.

Definition ((κ, μ, σ^2) – Model Set). We are given the first three moments m_0, m_1, m_2 of the process impulse response (\cdot) and fixed $n \in \{2, \dots, \infty\}$. A transfer function (\cdot) is an element of the plant family $\mathbb{R}^n(\kappa, \mu, \sigma^2)$ if it is consistent with the two following conditions:

(i) (A priori Hypothesis)

$$(\cdot) = \frac{1}{p(s)}$$

where (\cdot) , $\deg((\cdot)) \leq n$, is a polynomial with real nonnegative coefficients, and all roots of $p(s)$ lie in the interval $(-\infty, 0]$.

(ii) (Experimental Data)

$$m_i = \int_0^\infty t^i (\cdot) dt = 0 \quad i = 0, 1, 2,$$

$$\kappa = m_0,$$

$$\mu = m_1 / m_0,$$

$$\sigma^2 = m_2 / m_1 - m_1^2 / m_0^2.$$

PID H_∞ Designer

Input :

Model Set:

$$S_{\mathbb{R}^+}^n(\kappa, \mu, \sigma^2),$$

$$n = 20, \kappa = 1, \mu = 1, \sigma^2 = 0.6$$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$$M_s \leq 1.6, M_T \leq 1.1$$

Output :

$$k_p = 2.587$$

$$k_i = 4.311$$

$$k_d = 0.25$$

$$b = 0.8, c = 1$$

