MILOŠ SCHLEGEL - MICHAL BRABEC

PID H_∞ DESIGNER

INTRODUCTION

REX CONTROLS

www.pidlab.com

Contents

PID design method

- For a long time, the development of PID controller design methods has been the goal of the control community. Despite that manual model-free tuning of controllers is still the most commonly used PID design method in industry.
- Tuning rules (Ziegler-Nichols, Lambda tuning, AMIGO method [1], Internal model control, Skogestad's SIMC method [2], …)

  Universal relations between model and controller parameters.

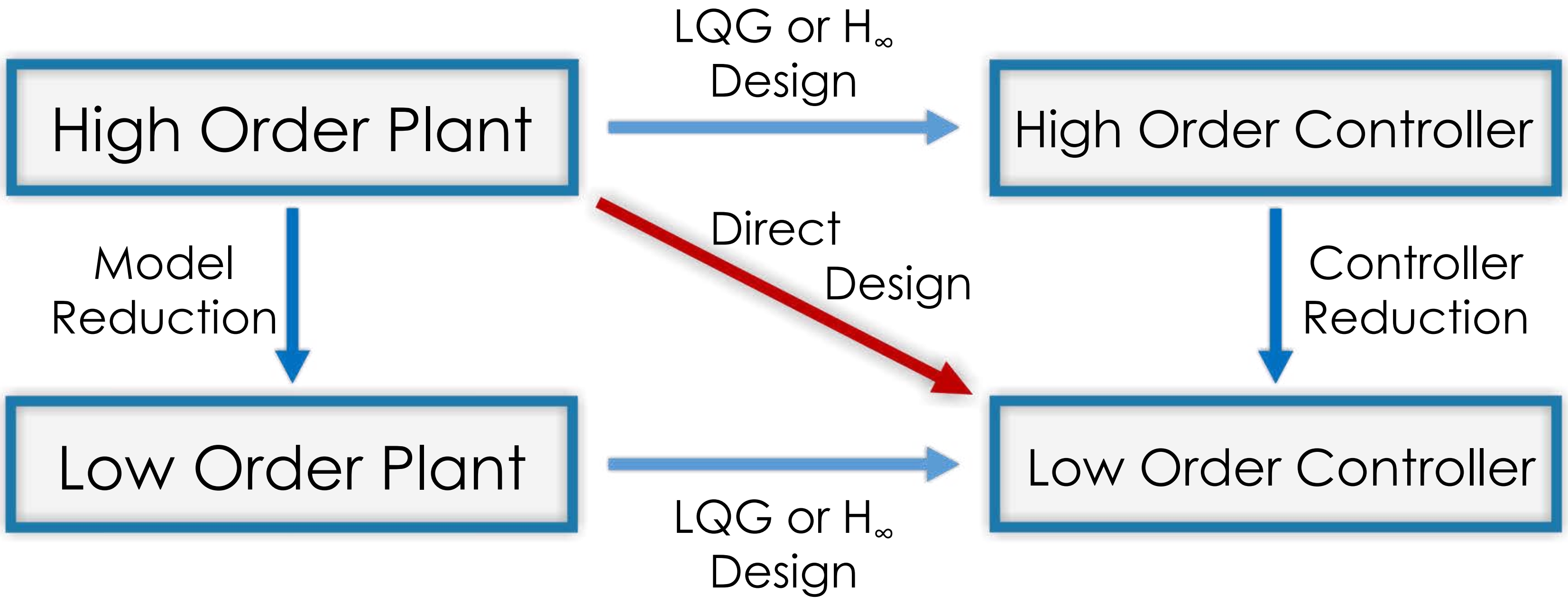
- Optimization-based method (MIGO [3], SWORD [4], MATLAB pidTuner)

  Treats each process model individually.

[1] Astrom, K.J. and Hagglund, T.: *Advanced PID Control*. ISA, 2006, ISBN 1-55617-942-1
[2] Skogestad, S. and Grimholt, Ch.: *The SIMC Method for Smooth PIDController Tuning*. PIDControl in the Third Millennium.Springer. 2012
[3] Astrom, K.J., Panagopoulos, H., Hagglund, T.: *Design of PI Controllers based on Non-Convex Optimalization*. Automatica, Vol. 34, No. 5, pp. 585-601, 1998.
[4] Garpinger, O.: *Analysis and Design of Software-Based Optimal PID Controllers*. PhD Thesis, Department of Automatic Control Lund University, 2015.

There exists no generally accepted design method for PID controller

The design procedures associated with modern control theory (H_∞ , LQG) provide high order controllers. Practice prefers simple controllers.



Anderson, B.D.O.: *Controller Design Moving from Theory to Practice*. 1992 Bode Prize Lecture.

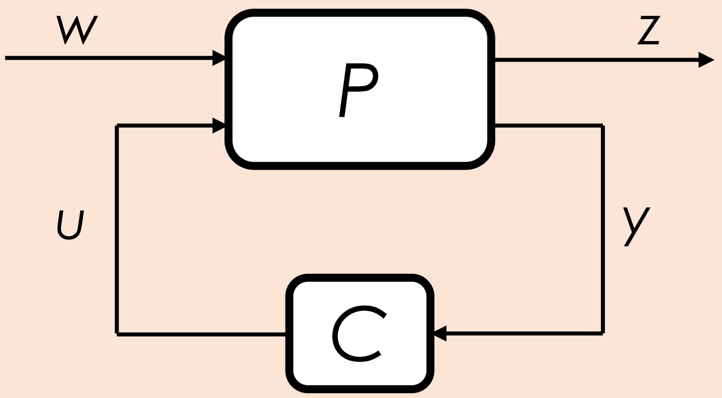
Requirements for effective design method

- **Versatility**: It should be applicable to a wide range of systems (i.e. stable/unstable/non minimal phase/oscillatory process transfer functions)

- **Adaptability/Practicality** : It should have the possibility to introduce specifications that capture the essence of real control problems (i.e. robustness/performance trade-off, servo/regulator problem)

- **Clear answer**: The method should be robust in the sense that it provides controller parameters if they exist, or if the specifications cannot be meet an appropriate diagnosis should be presented

The general H∞ Control Problem



minimize $\left\| H_{w \to z}(P, C) \right\|_\infty$

subject to C stabilizes P internally

$C \in \mathbf{C}$

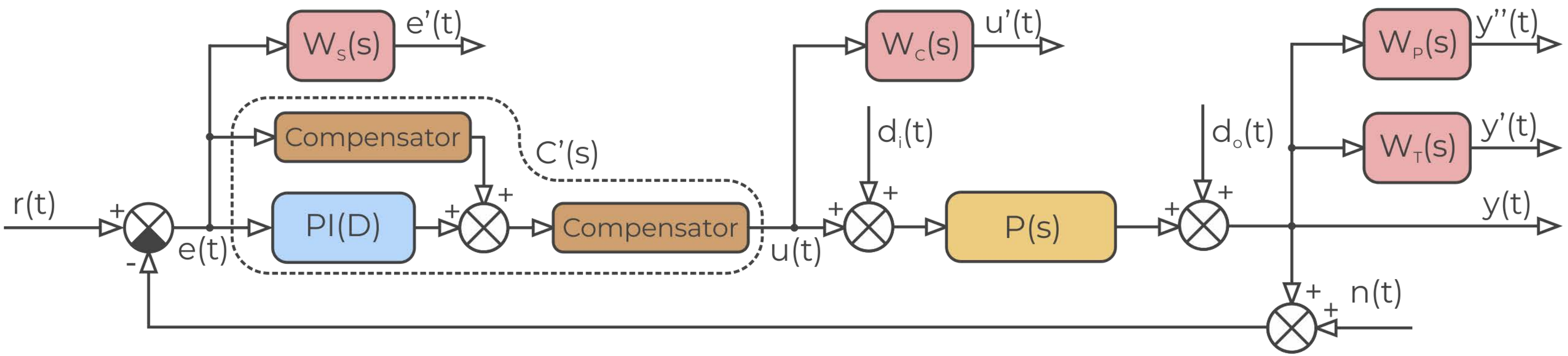
$P = P(s)$	Given a real rational transfer matrix called the plant
$C \in \mathbf{C}$	Searched controller from the controller space $\mathbf{C}$
$H_{w \to z}(P, C)$	The closed-loop performance or robustness transfer matrix

In our considered case, $H \equiv H_{w \to z}(P, C)$ is a scalar function and it holds

$$\left\| H \right\|_\infty \triangleq \sup_{w \neq 0} \frac{\left\| Hw \right\|_2}{\left\| w \right\|_2} = \sup_{w \neq 0} \frac{\left\| z \right\|_2}{\left\| w \right\|_2} = \max_{\omega} \bar{\sigma}\left(H(j\omega) \right)$$

$$\left\| z \right\|_2 \triangleq \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} Tr\left(H(j\omega) H^H(j\omega) d\omega \right) \right)^{\frac{1}{2}}$$

The H∞ Control Problem considered



Find all controllers C' for which it holds

$$\left\| H_{w \to z}(P, C') \right\|_\infty \leq \gamma$$

subject to C' stabilizes P internally

$$C' \triangleq (C_{PID} + C_{comp}) \cdot C_{comp} \in \mathbf{C}.$$

The performance or robustness channel $H \equiv H_{w \to z}(P, C)$ is a scalar weighting closed-loop sensitivity function and it holds

$$\|H\|_\infty \triangleq \sup_{w \neq 0} \frac{\|Hw\|_2}{\|w\|_2} = \sup_{w \neq 0} \frac{\|z\|_2}{\|w\|_2} = \max_\omega |H(j\omega)|$$

$$\|z\|_2 \triangleq \left(\int_{-\infty}^{+\infty} z^2(t)\, dt \right)^{\frac{1}{2}} = \left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} |Z(j\omega)|^2 d\omega \right)^{\frac{1}{2}}$$

REX CONTROLS

www.pidlab.com

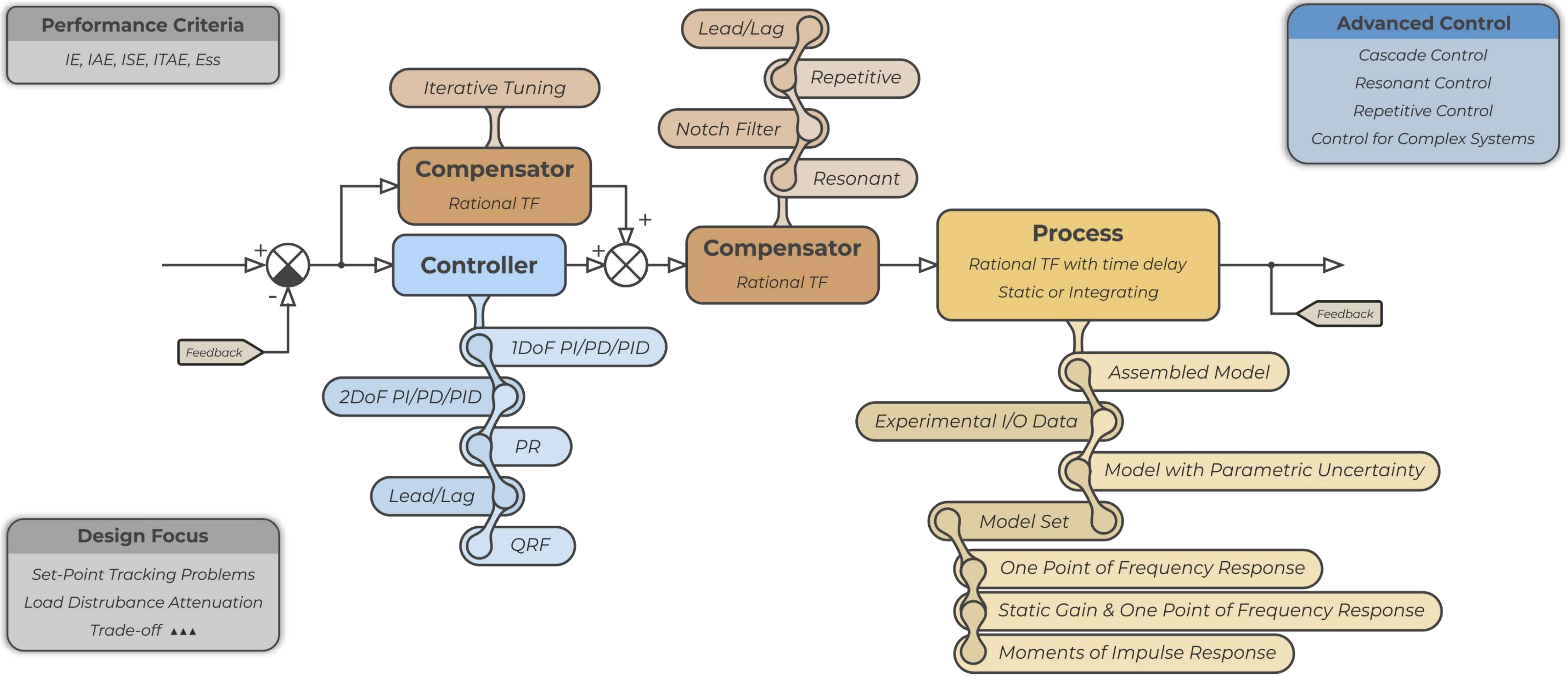
PID H_∞ Designer (1)

- PID H_∞ Designer is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE, ISE and H_∞ robustness constraints.
- PID H_∞ Designer can be used for a wide range of process models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, ...) and also for so-called model sets created from any number of process transfer functions.
- Supported design specifications reflect the essence of real control problems. Optimization of integral criteria IE, ISE, IAE, ITAE under H_∞ constraints is supported for both load disturbance attenuation and set-point tracking problems).
- Designing of PI(D) controller with typical specifications using PID H_∞ Designer is a routine procedure that does not require deeper knowledge of control theory from the user.

PID H_∞ Designer (2)

- With more skills and efforts from the designer it should be possible using PID H_∞ Designer to design high performance PID controllers extended with a suitable linear compensator (Cascade Controller, Resonant Controller, Smith Predictor, Repetitive Control, ...).
- PID H_∞ Designer also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one point of the frequency response). Moreover, the non-standard moment model set provided by the PIDMA-autotuner from the company REX Controls is also supported.

PID H_∞ Designer: Options



PID H_∞ Designer: Design Environment

The user can choose between two design environments. Each of them is specifically designed to accommodate users with different levels of expertise.

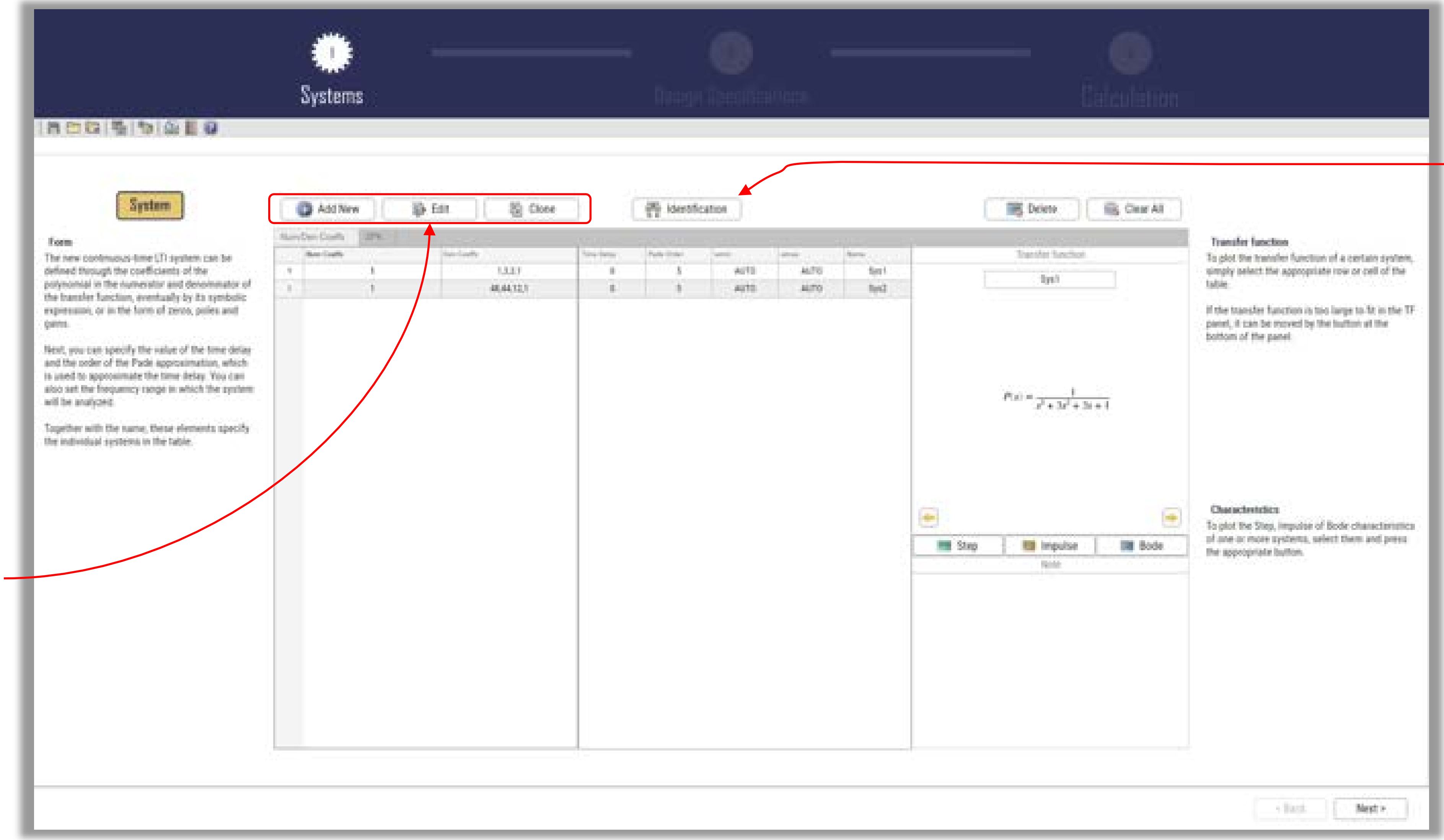
Step By Step

The environment is intended primarily for beginners who are working with the tool for the first time. For these reasons, the design process is divided into several steps (slides). All necessary information is then explained in the individual phases of the design process.

WorkSpace

The environment is more suitable for advanced users who are already familiar with the design process. This environment also includes a set of auxiliary functions and settings to streamline the design and analysis of the solution.

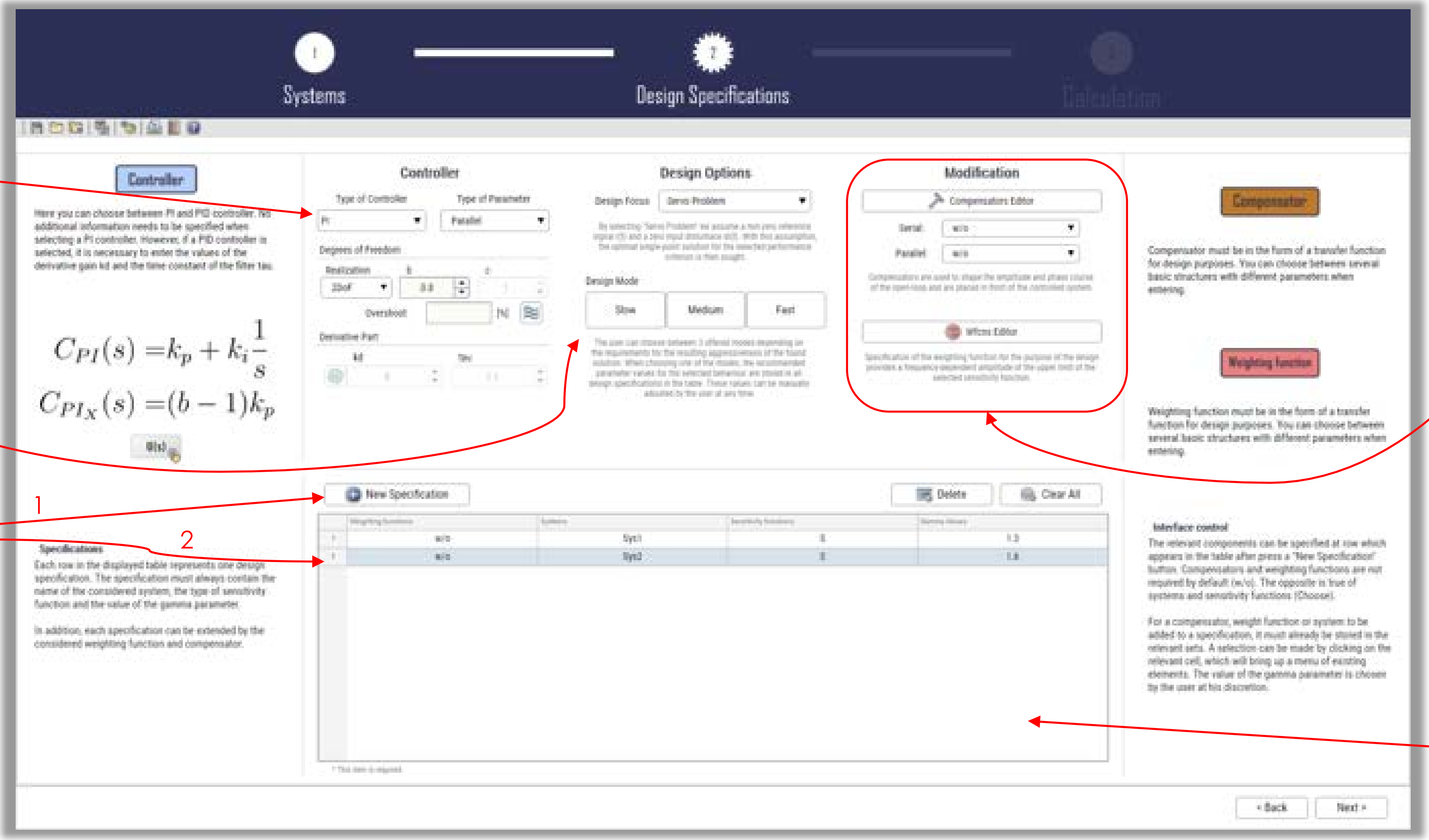
PID H_∞ Designer GUI: Step By Step (1)



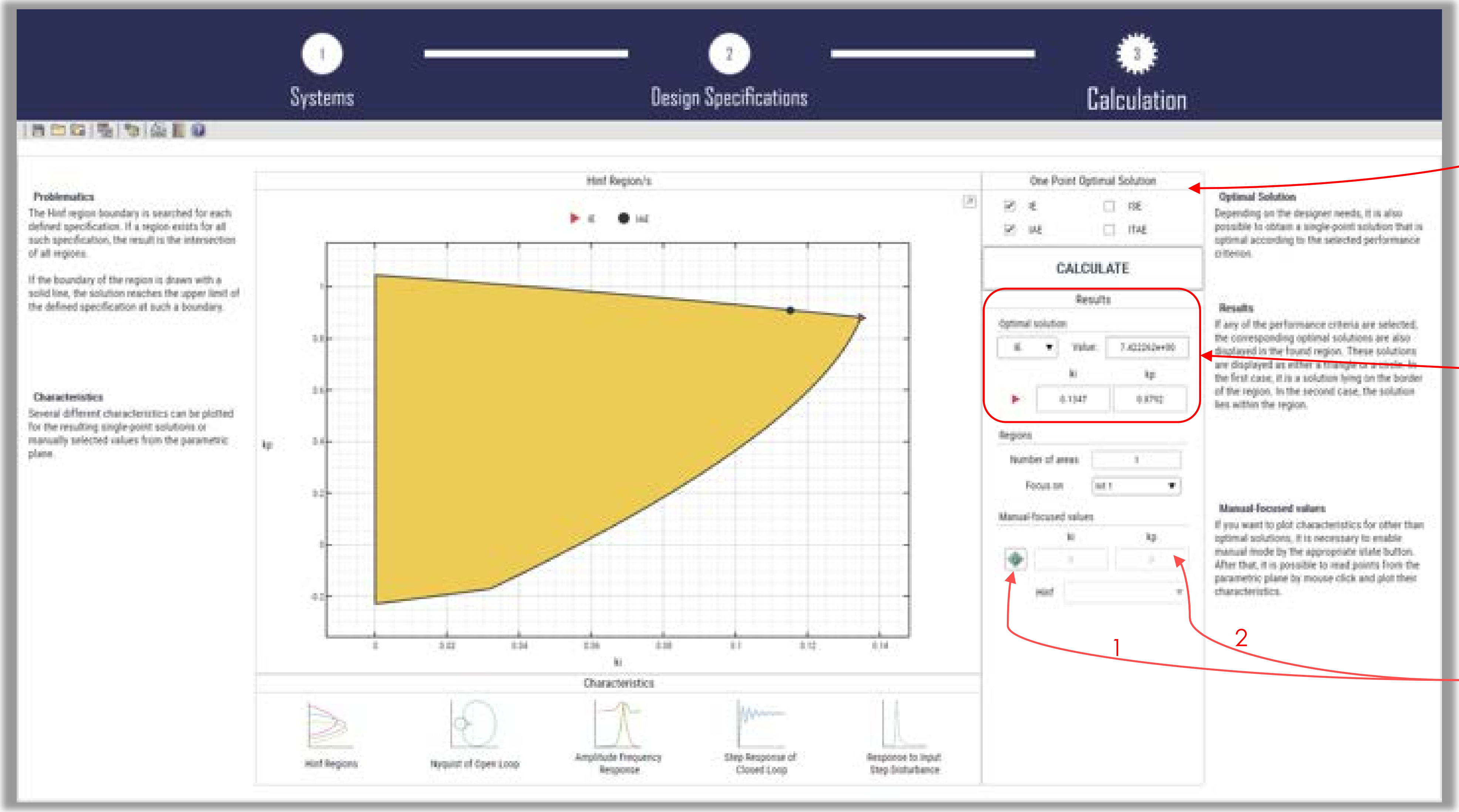
System Identification Experimental I/O data (See Appendix E)

Entering rational transfer function + time delay of processes

PID H∞ Designer GUI: Step By Step (2)



PID H_∞ Designer GUI: Step By Step (3)



Selection of the criterion function

The resulting controller

Manual tuning of the controller

PID H_∞ Designer GUI: WorkSpace

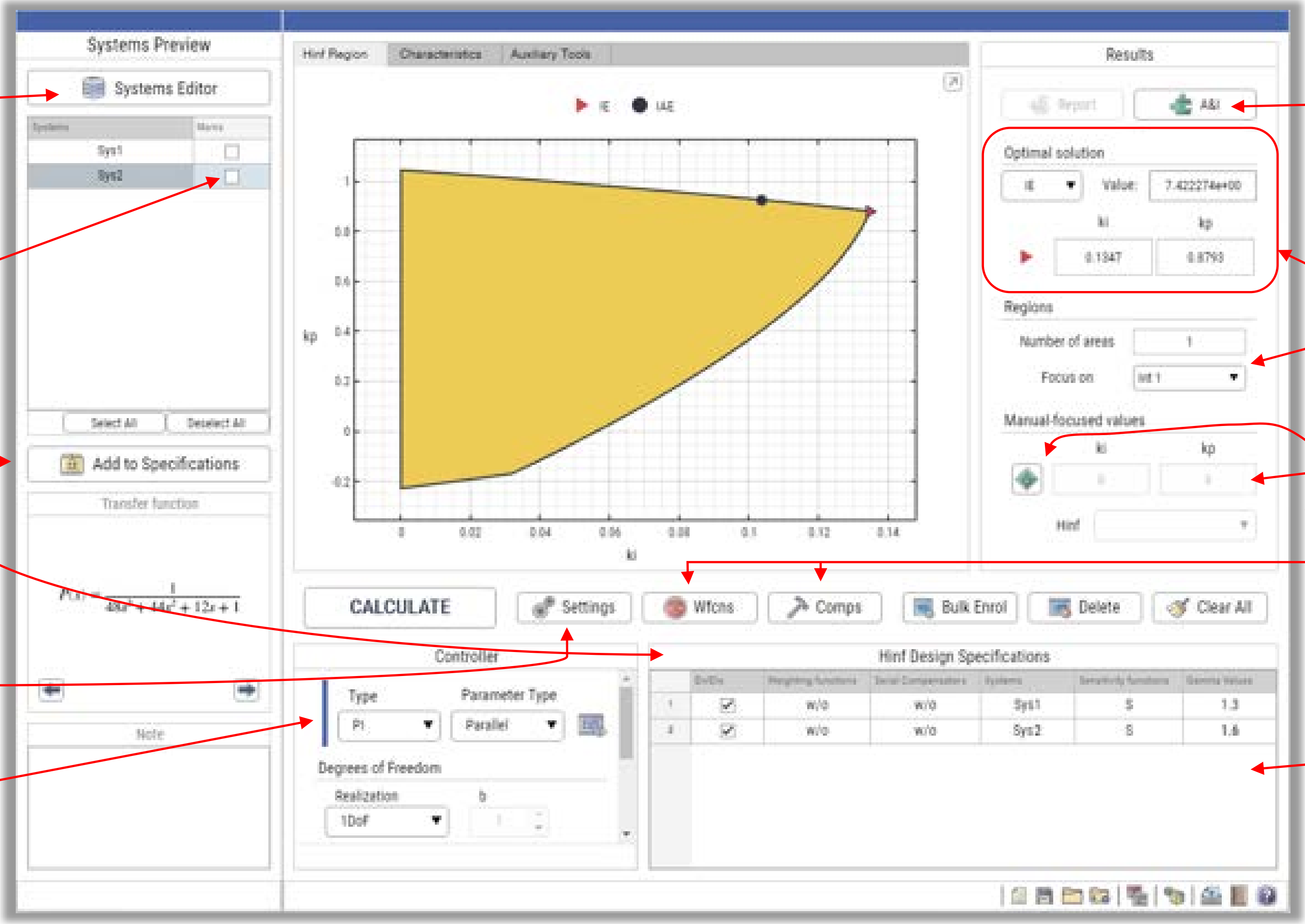
Entering transfer functions of processes

Selection of a model set

1 Enter H_∞ limitations 2

Selection of the design criterion

Select the controller type



Create a closed-loop assembled transfer function (e.g. for cascade control) or store actual controller for iterative modification via parallel compensator

The resulting controller

H_∞ region selection

1 Manual tuning of the controller 2

Enter weighting functions and compensators

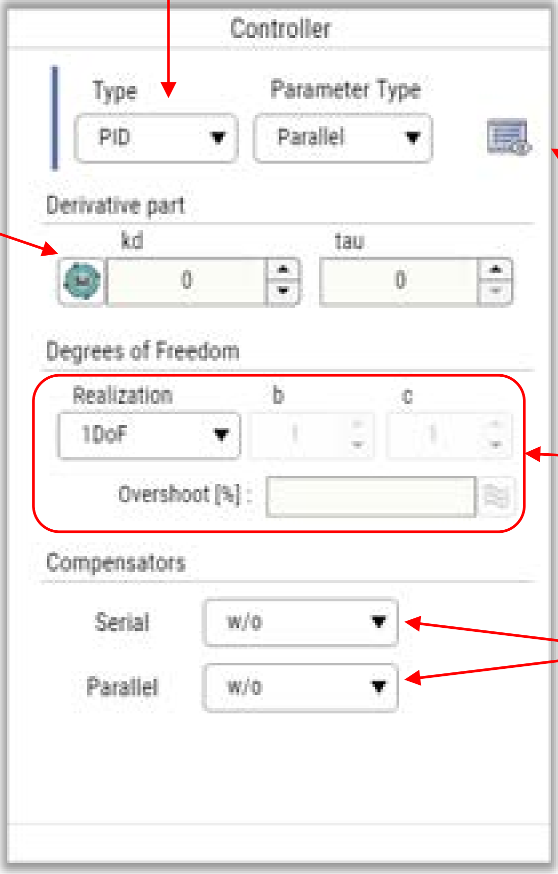
Select weighting functions, compensators, systems, sensitivity functions and values of H_∞ limitations

PID H_∞ Designer GUI: WorkSpace

Controller Settings

Affine-structured controller labelled as QRF → Conclusion (2)

Estimate kd value

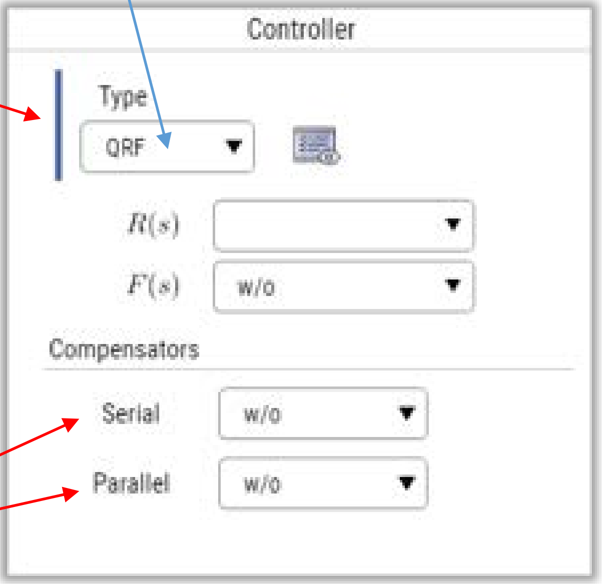


Select the controller type

Show controller form

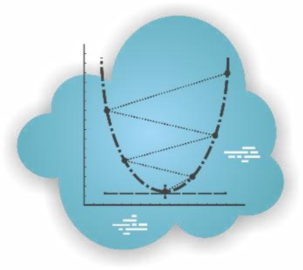
Manage DoF

Choose compensators



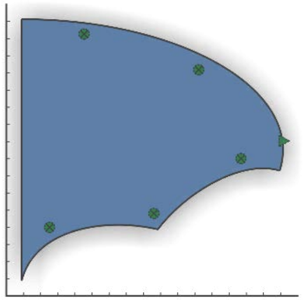
PID H_∞ Designer GUI: WorkSpace

Auxiliary Tools (1)



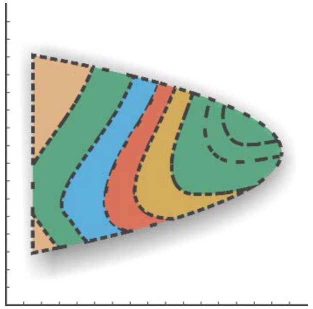
Find the Minimum Gamma Value

Finding the minimum gamma value for selected design constraints



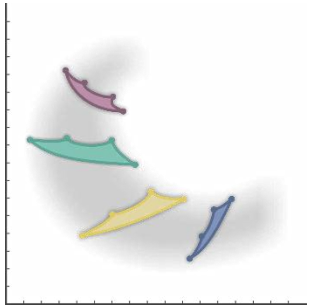
Multipoint Analysis

Analysis of the obtained optimal solutions with possibly of exploration over the H_∞ region to achieve the desired behavior



Performance Criteria Contour Line

Computing contour lines of selected criterion function for solutions from the H_∞ region

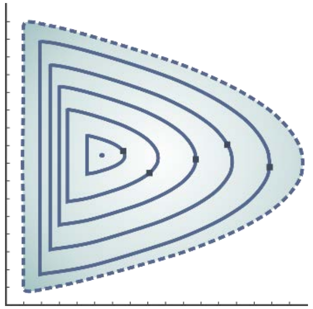


Open Loop-Value Set Region/s

Show open loop value set/s for selected frequencies. Value set/s represents a model uncertainty on these frequencies

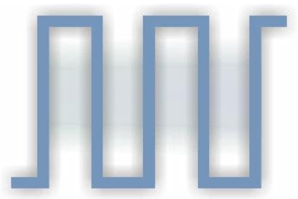
PID H_∞ Designer GUI: WorkSpace

Auxiliary Tools (2)



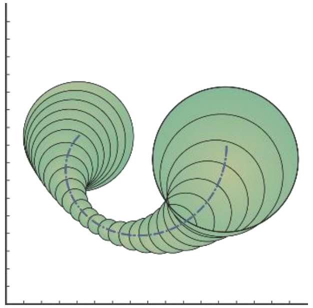
Multiparametric Analysis

Analysis of the choice of the parameter gamma, k_d, or τ according to several characteristics



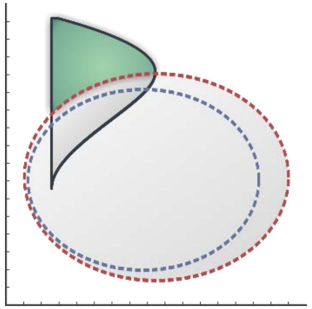
Signal Response

Simulate system responses for various types of signals.



Tolerance Circles

Calculation of tolerance circles of nominal open loop frequency characteristic in complex plane according to selected nominal gamma value and maximal gamma value



ε – Constraint

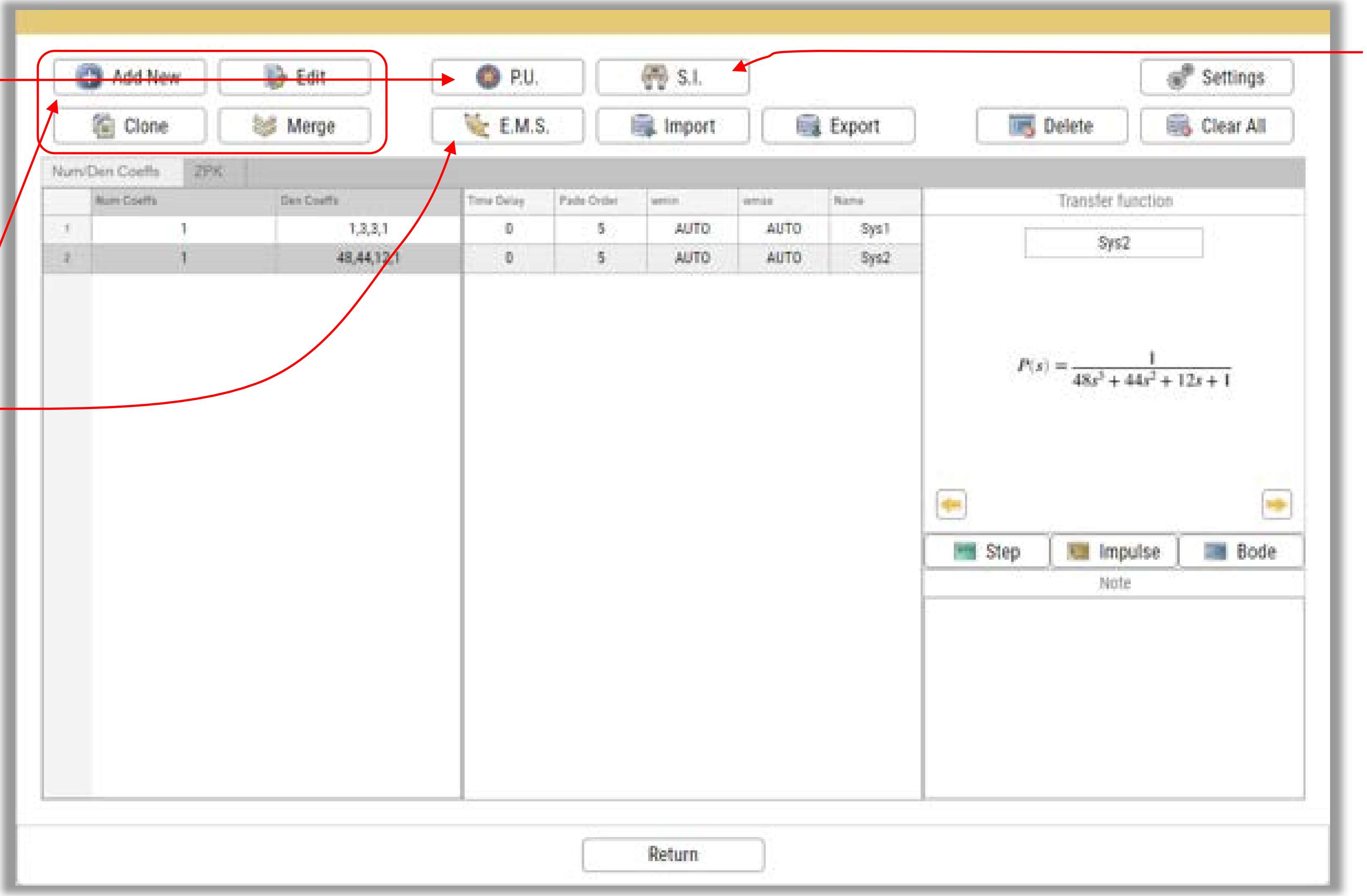
Find the solution that satisfy selected amplitude limit ε on the frequency interval between ω_1 and ω_2

PID H_∞ Designer GUI – Systems Editor

Parameter Uncertainty Model Set (See Appendix C)

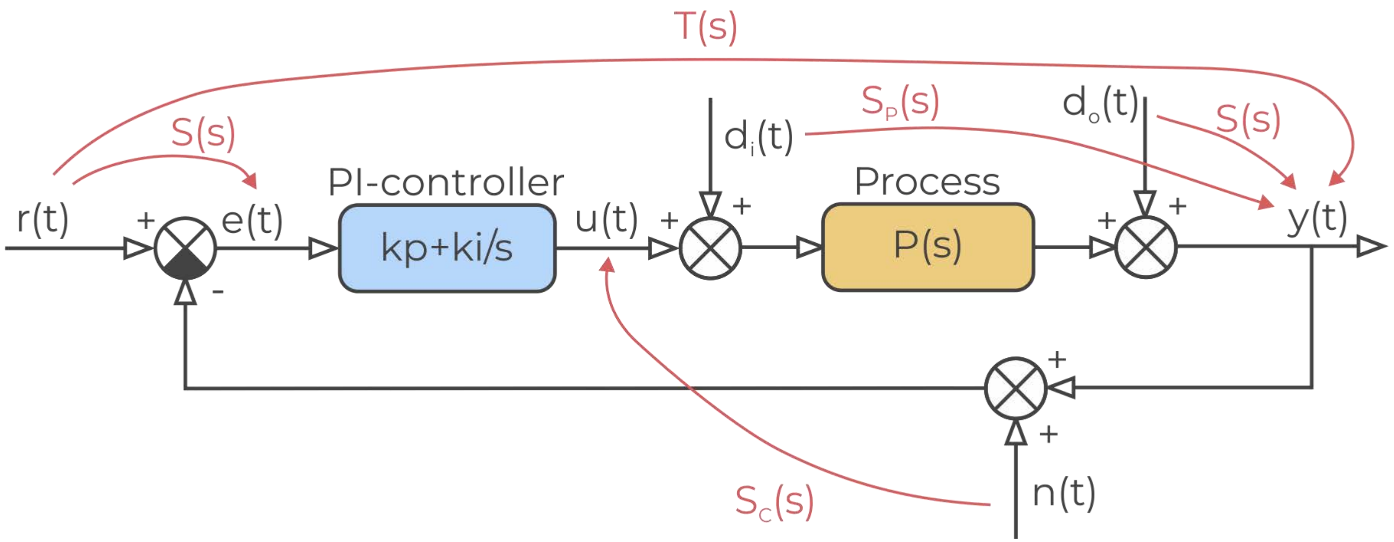
Rational Transfer Function + Time Delay

Experimentally Determined Model Set (See Appendix D)



System Identification Experimental I/O data (See Appendix E)

Parameter Plane Formulation of Basic PI-Controller Design Problem



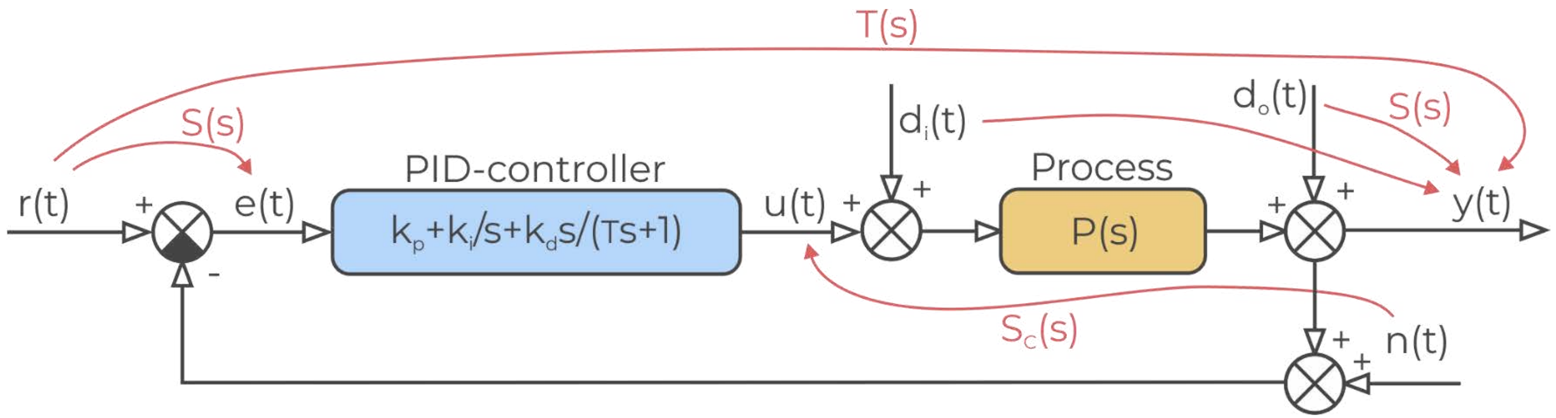
$H(s,k=[k_i,k_p]) \triangleq W(s)S_*(s,k),\;\; S_* \in \{S,T,S_C,S_P\}$ .... weighting sensitivity function

$\mathrm{K} \;\; \{[\;_i,\;_p]: \|\;(\;,\;)\|_\infty \;\; \sup_\omega|\;(\;\omega,\;)| \le \gamma,$ the closed-loop is stable$\}$ $_\infty$-region in the parameter plane

1) Find the H_∞- region K in the k_i-k_p plane. (See Appendix A for details.)

2) Find the optimal PI-controller in the H_∞- region K with respect to the criterion $IAE \triangleq \int_0^\infty |e(t)|dt$ for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

Parameter Plane Formulation of Basic PID-Controller Design Problem



$H(s,k=[k_p,k_i,k_d,\tau]) \triangleq W(s)S_*(s,k),\ \ S_* \in \{S,T,S_C,S_P\}$ .... weighting sensitivity function

$\mathrm{K}_{[\ _d,\tau]} \triangleq \Big\{[\qquad]\ \Vert\qquad\Vert_\infty \triangleq \ _{\omega}\ |\quad \omega\quad | \leq \gamma$ the closed-loop is stable$\Big\}$ ... $_\infty$ region in the parameter plane

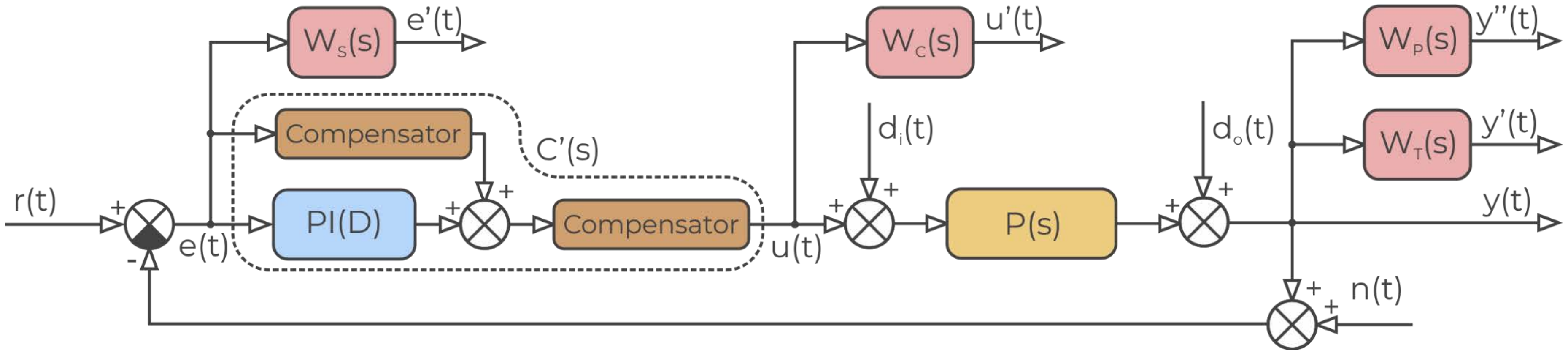
for the fixed k_d and τ

1) Choose the derivative gain k_d and the time constant τ manually or with the help of a built-in function. (See Appendix B for details.)

2) Find the H_∞ - region $\mathrm{K}_{[k_d,\tau]}$ in the k_i - k_p plane.

3) Find the optimal PID-controller in the H_∞ region $\mathrm{K}_{[k_d,\tau]}$ with respect to the criterion $IAE \triangleq \int_0^\infty |e\ t|dt$ for the step in the reference value r (servo problem) or load disturbance d_i (regulator problem).

H_∞ limitations supported



$$\|H(s)\|_\infty \le \gamma \Leftrightarrow |H(j\omega)| \le \gamma, \forall \omega \in [0, \infty)$$

Sensitivity functions (gang of four)

$$S = \frac{1}{1 + C'P}, \; T = C'PS, \; S_C = C'S, \; S_P = PS$$

Weighting functions

$$W_S(s), W_T(s), W_C(s), W_P(s),$$

Servo problem (set-point tracking)

$$r \to e' : \; \|W_S S\|_\infty \le M_S$$

$$r \to y' : \; \|W_T T\|_\infty \le M_T$$

$$n \to u' : \; \|W_C S_C\|_\infty \le M_C$$

Regulator problem (load disturbance rejection)

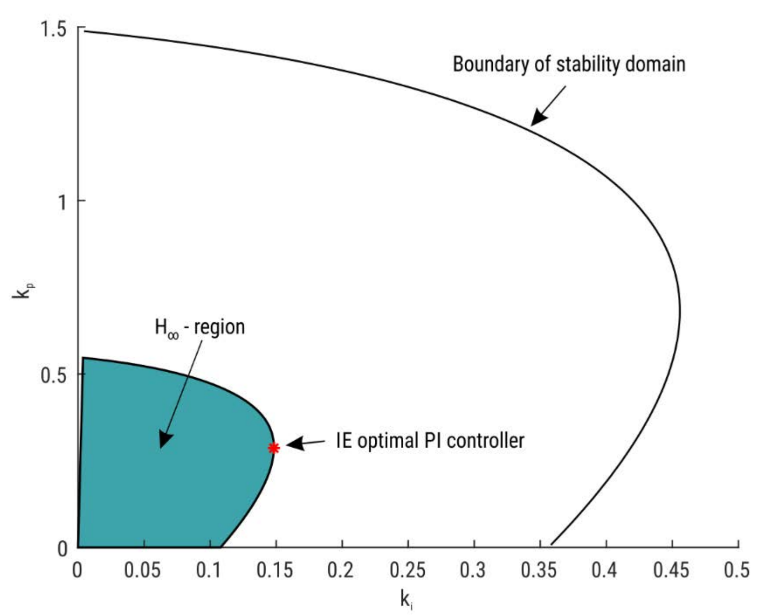
$$d_i \to y'' : \; \|W_P S_P\|_\infty \le M_P$$

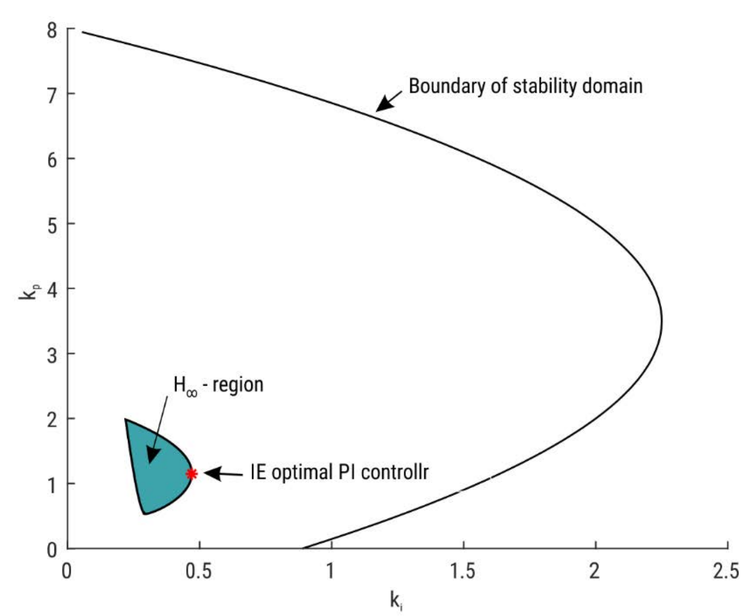
$$d_o \to e' : \; \|W_S S\|_\infty \le M_T$$

$$n \to u' : \; \|W_C S_C\|_\infty \le M_C$$

H_∞-Region in the Parametric Plane $k_i - k_p$

(It contains all PI controllers that meet the specified H_∞ limitations)





Finding the H_∞ - region

$$\mathrm{K} \triangleq \left\{k = \left[k_i, k_p\right] : \left\|H(s,k)\right\|_\infty \leq \gamma, \text{ the closed-loop is stable}\right\}$$

is generally a very difficult problem. PID Hinf Designer (www.pidlab.com) is the first software tool available to fully address this issue.

Example of Simple Design specification of PI-controller for FOPDT system

Proces transfer function: $P(s) = \dfrac{e^{-s}}{s+1}$

Controller transfer function: $C_{PI}(s) = K\left(1 + \dfrac{1}{T_i s}\right) = k_p + \dfrac{k_i}{s}$

Sensitivity function: $S(s) = \dfrac{1}{1 + C_{PI}(s)P(s)}$

Weighting function: $W_S(s) = 1$

Type of control problem: regulator problem (load step disturbance rejection)

Design specification: $IAE = \min\limits_{C_{PI}} \int_0^\infty \left|e(t)\right| dt$

subject to $\left\|S(s)\right\|_\infty \leq M_S \quad \Leftrightarrow \quad \left|S(j\omega)\right| \leq M_S,\ \forall \omega \in [0, \infty)$

PID H_∞ Designer

Input :

$$P(s) = \frac{e^{-s}}{s+1}$$

PI-controller $\left(C_{PI}(s) = k_p + \frac{k_i}{s} \right)$

$$M_S = 1.6$$

Output :

$IE: \quad k_p = 0.463, \quad k_i = 0.509$

$IAE: \quad k_p = 0.565, \quad k_i = 0.488$

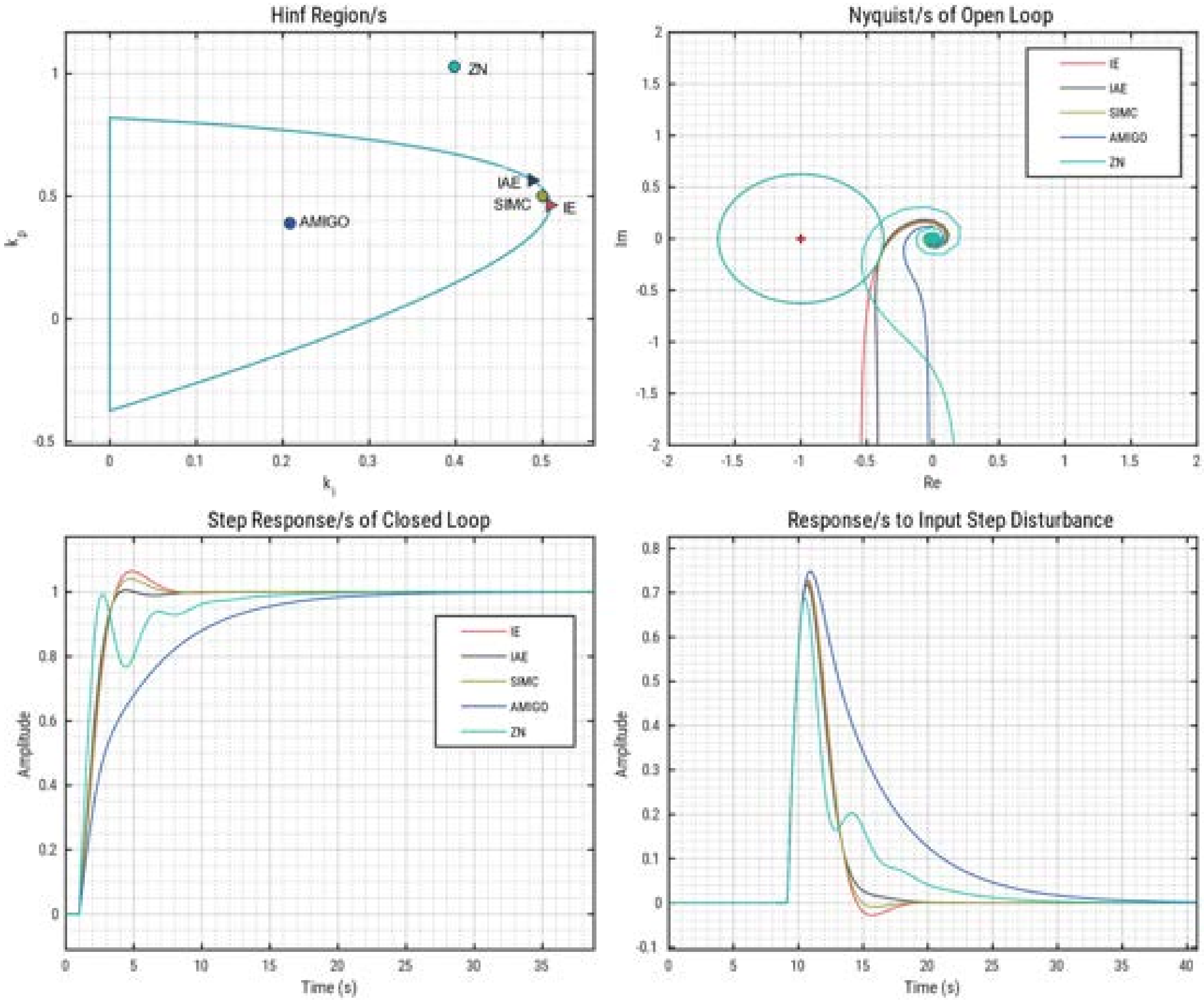
$ITAE: \quad k_p = 0.557, \quad k_i = 0.492$

Optional output :

ZN Ziegler-Nicols (1942, step response)

SIMC Skogestad (2012)

AMIGO Hagglund and Astrom (2004)



More General Formulation of Design Problem

(fully supported by PID H_∞ Designer)

$\mathbf{P} = \{P_1, P_2, \ldots, P_k\}$ model set of transfer functions

$P \in \mathbf{P}$ process transfer function

$C \in \{C_{PI}, C_{PID}\}$ controller transfer function

$S = \dfrac{1}{1+CP},\ \ T = \dfrac{CP}{1+CP},\ \ S_C = \dfrac{CP}{1+CP},\ \ S_P = \dfrac{P}{1+CP}$ loop sensitivity transfer functions

$\mathbf{I} = \{IE, IAE, ITAE, ISE\}$ design criterion set

$$IE \triangleq \int_0^\infty e(t)dt,\ \ IAE \triangleq \int_0^\infty |e(t)|\,dt,\ \ ITAE \triangleq \int_0^\infty t\,|e(t)|\,dt,\ \ ISE \triangleq \int_0^\infty e^2(t)dt$$

$I \in \mathbf{I}$ design criterion selected

W_S, W_T, W_C, W_P weighting functions

Controller Robust Design Problem

$$\min_C \max_{P \in \mathbf{P}} I$$

subject to the H_∞ limitations

$$\forall P \in \mathbf{P}:\ \ \|W_S S\|_\infty \le M_S,\ \|W_T T\|_\infty \le M_S,\ \|W_C S_C\|_\infty \le M_C,\ \|W_P S_P\|_\infty \le M_P.$$

Example of Design Specification of Robust PI-controller for Process Model Set

Proces model set: $\mathbf{P} \triangleq \left\{ P_1(s) = \dfrac{-0.0216s + 0.0031}{s^2 + 0.457s + 0.0868} e^{-0.166s},\ P_2(s) = \dfrac{-0.0174s + 0.0046}{s^2 + 0.5978s + 0.0445} e^{-0.166s} \right\}$

Controller transfer function: $C_{PI}(s) = K\left(1 + \dfrac{1}{T_i s}\right) = k_p + \dfrac{k_i}{s}$

Sensitivity functions: $S_i(s) = \dfrac{1}{1 + C_{PI}(s)P_i(s)},\ i = 1,2$

Weighting functions: $W_i(s) = 1,\ i = 1,2$

Type of control problem: regulator problem (load step disturbance rejection)

Design specification: $\min_{C_{PI}} \max_{i \in \{1,2\}} \int_0^\infty |e_i(t)|\, dt$

subject to $\|S_i(s)\|_\infty \le M_S,\ i = 1,\ldots,2 \iff |S_i(j\omega)| \le M_S,\ i = 1,2,\ \forall \omega \in [0,\infty)$

PID H_∞ Designer

Input :

$$P_1(s) = \frac{-0.0216s + 0.031}{s^2 + 0.457s + 0.0868} e^{-0.166s}$$

$$P_2(s) = \frac{-0.0174s + 0.0445}{s^2 + 0.5978s + 0.0445} e^{-0.166s}$$

$$C_{PI}(s) = k_p + \frac{k_i}{s}$$

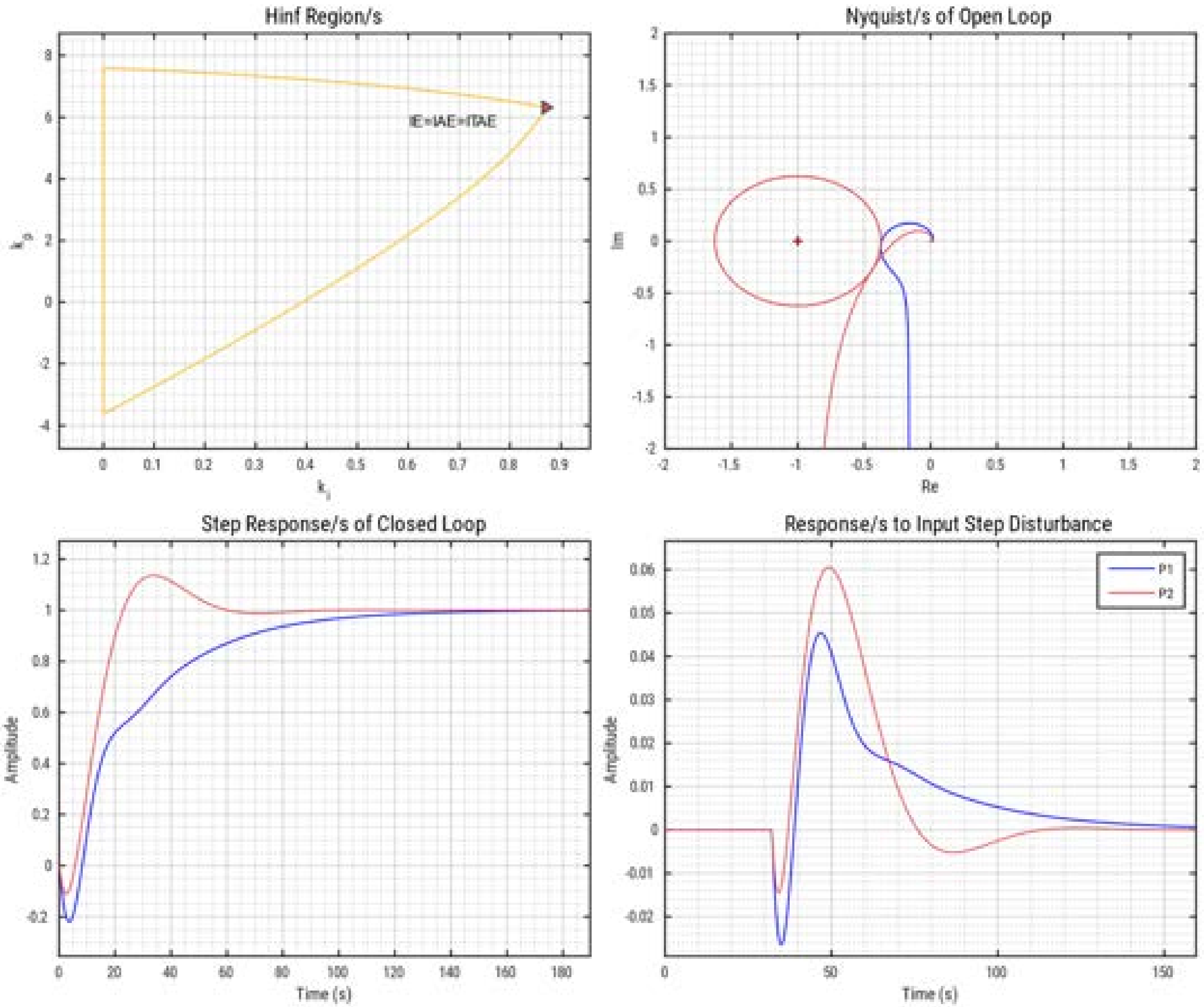
$$M_S = 1.6$$

Output :

$$IE: \quad k_p = 6.313, \quad k_i = 0.8704$$

$$IAE: \quad k_p = 6.313, \quad k_i = 0.8704$$

$$ITAE: \quad k_p = 6.313, \quad k_i = 0.8704$$



Conclusion (1)

- PID H_∞ Designer is the first advanced easy to used web design tool for the analysis and design of optimal PI(D) controllers with respect to performance integral criteria IE, IAE, ITAE and H_∞ robustness constraints.
- PID H_∞ Designer can be used for a wide range of process models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, …) and also for so-called model sets created from any number of process transfer functions.
- PID H_∞ Designer provide a new explicit algorithm to determine the H_∞- regions in the parameter plane of PI controller for all commonly used H_∞ limitations of the weighted sensitivity functions.
- PID H_∞ Designer also supports simple process models obtained from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one frequency point). Moreover, the non-standard moment model set provided by the PIDMA-autotuner from the company REX Controls is also supported.

Conclusion (2)

- Designing of PI(D) controller with typical specifications using PID H_∞ Designer is a routine procedure that does not require deeper knowledge of control theory from the user.
- With more skills and efforts from the designer it should be possible to design high performance PID controllers extended with any linear compensator suitable (Resonant Controller, Smith predictor, Repetitive Control, …).
- More details about the affine-structured controller in Semi-Plenary Lecture or in white paper „*Analytical Design of a Wide Class of Controllers with Two Tunable Parameters Based on H_∞ Specifications*"

Semi-Plenary Lecture
Process Control 2023

H_∞ Affine Controller
White Paper

Appendix A: Isolation of H_∞-Region (1)

For more details see: Schlegel M., Medvecová P., *Design of PI Controllers : H_{inf} Region Approach*. IFAC PapersOnLine 51-6 (2018), 13-17.

Proposition : If $C(s,k) = k_p + \frac{k_i}{s}$, $k = [k_i, k_p]$, $P(s)$ has no poles on the imaginary axis, and the design specification is

$$\|S(s,k)\|_\infty = \left\|\frac{1}{1+C(s,k)P(s)}\right\|_\infty = \left\|\frac{S_n(s,k)}{S_d(s,k)}\right\|_\infty \le \gamma \triangleq M_S \ne 1,$$

then the boundary of the H_∞-region K is contained in the solutions of the two systems of equations

(i) $$S_n(j\omega,k) = 0, \quad S_d(j\omega,k) = 0,$$

(ii) $$|S(j\omega,k)|^2 = \gamma^2, \quad \frac{\partial |S(j\omega,k)|^2}{\partial \omega} = 0.$$

The system of equations (i) has a solution $k_i = 0$, i.e. any point on the axis k_p is a solution of this system. The solution of the system (ii) is determined by the parametric curves

Appendix A (2)

$$\left.\begin{aligned} k_i &= \frac{x_i \omega}{M_S}, \\ k_p &= \frac{{}^2 \quad {}^2 + \quad {}^2\,{}^2 + \qquad \omega - \quad {}^2{}_1 + 2 \quad {}_1 + \quad {}^2{}_1 + \quad {}^2 + \quad {}^2 + \omega \quad {}^2 - 1 \quad {}_1 + \quad {}_1}{\omega M_S^2 (2ABB_1 + A^2 A_1 - A_1 B^2)}, \end{aligned}\right\} \quad \omega \in [0, \infty),$$

where A, A_1, B, B_1 are the functions of ω defined by

$$P(j\omega) = A(\omega) + jB(\omega) \triangleq A + jB,$$

$$\frac{dP(j\omega)}{d\omega} = A_1(\omega) + jB_1(\omega) \triangleq A_1 + B_1,$$

and $x_i, i = 1, \ldots, l(\omega),\ l(\omega) \in \{0, 2, 4\}$ are the frequency dependent real roots of quartic polynomial

$$a\,x^4 + b\,x^3 + c\,x^2 + d\,x + e = 0$$

with the real frequency dependent coefficients

$$a = (A^2 + B^2)^4,$$

$$b = 2M_S (A^2 + B^2)^2 (-\omega B_1 A^2 + 2\omega BAA_1 + \omega B^2 B_1 + BA^2 + B^3),$$

$$= -(\ {}^2 + \ {}^2)(2\omega \ {}_1 \ {}^3 + 4\omega \ {}^2 \ {}_1 \ {}^2_S + 2\omega \ {}^2 \ {}_1 - \omega^2 \ {}^2 \ {}^2_1 \ {}^2_S - \omega^2 \ {}^2 \ {}^2_1 \ {}^2_S - \ {}^2 \ {}^2 \ {}^2_S - 8\omega \ {}_1 \ {}^2 \ {}^2_S +$$

$$+ 2\omega AA_1 B^2 - \omega^2 A_1^2 B^2 M_S^2 - B^4 M_S^2 + 2\omega B^3 B_1 - \omega^2 B^2 B_1^2 M_S^2 - 4\omega B^3 B_1 M_S^2),$$

$$d = -2\omega M_S (-\omega A_1^2 B^3 M_S^2 - \omega A^2 A_1^2 B M_S^2 - 2AA_1 B^3 M_S^2 - \omega A^2 B B_1^2 M_S^2 + A^2 B^2 B_1 M_S^2 - B^4 B_1 M_S^2 - \omega B^3 B_1^2 M_S^2 +$$

$$+ 3\omega \ {}_1 \ {}^2 \ {}_1 - \omega \ {}^2 \ {}^2_1 + \ {}_1 \ {}^3 + 2\omega \ {}^2 \ {}^2_1 + \omega \ {}^3 \ {}^2_1 - \omega \ {}^3 \ {}_1 \ {}_1 + \ {}^2 \ {}^2_1 + \ {}^3 \ {}_1 + \ {}^4 \ {}_1),$$

$$e = -\omega^2 (M_S^2 - 1)(-A_1^2 B^2 M_S^2 - B^2 B_1^2 M_S^2 + 2AA_1 BB_1 + B^2 B_1^2 + A^2 A_1^2).$$

Appendix A (3)

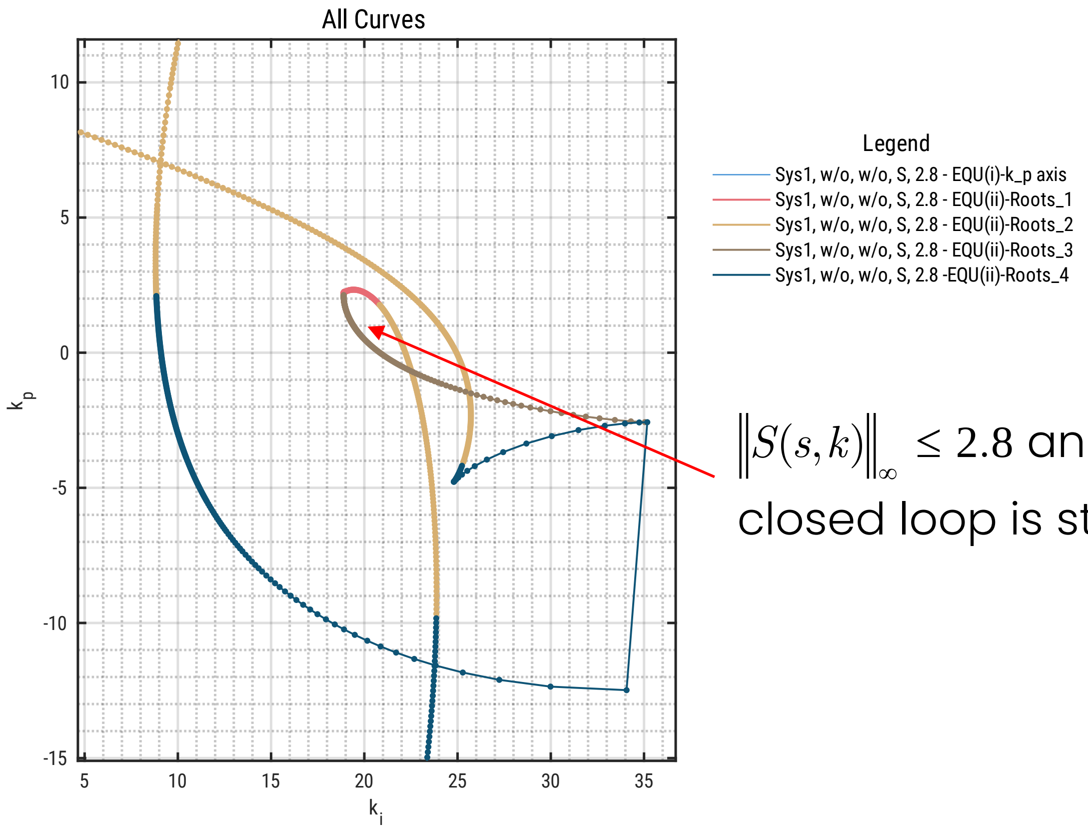
The curves representing the solutions of systems (i) and (ii) divide the parametric plane into regions. From them, it is necessary to select those that meet the design specifications. For this purpose, it is sufficient to test only one point of the respective region.

Example: H_∞ – region for unstable process:

$$P(s) = \frac{s^3 + 4s^2 - s + 1}{s^5 + 2s^4 + 32s^3 + 14s^2 - 4s + 50},$$

$$C(s,k) = k_p + \frac{k_i}{s}$$

$$\left\| S(s,k) \right\|_\infty = \left\| \frac{1}{1 + C(s,k)P(s)} \right\|_\infty \le M_S = 2.8$$



$\left\| S(s,k) \right\|_\infty \le 2.8$ and closed loop is stable

Appendix A (4)

WorkSpace → Auxiliary Tools →
→ Multiparametric Analysis

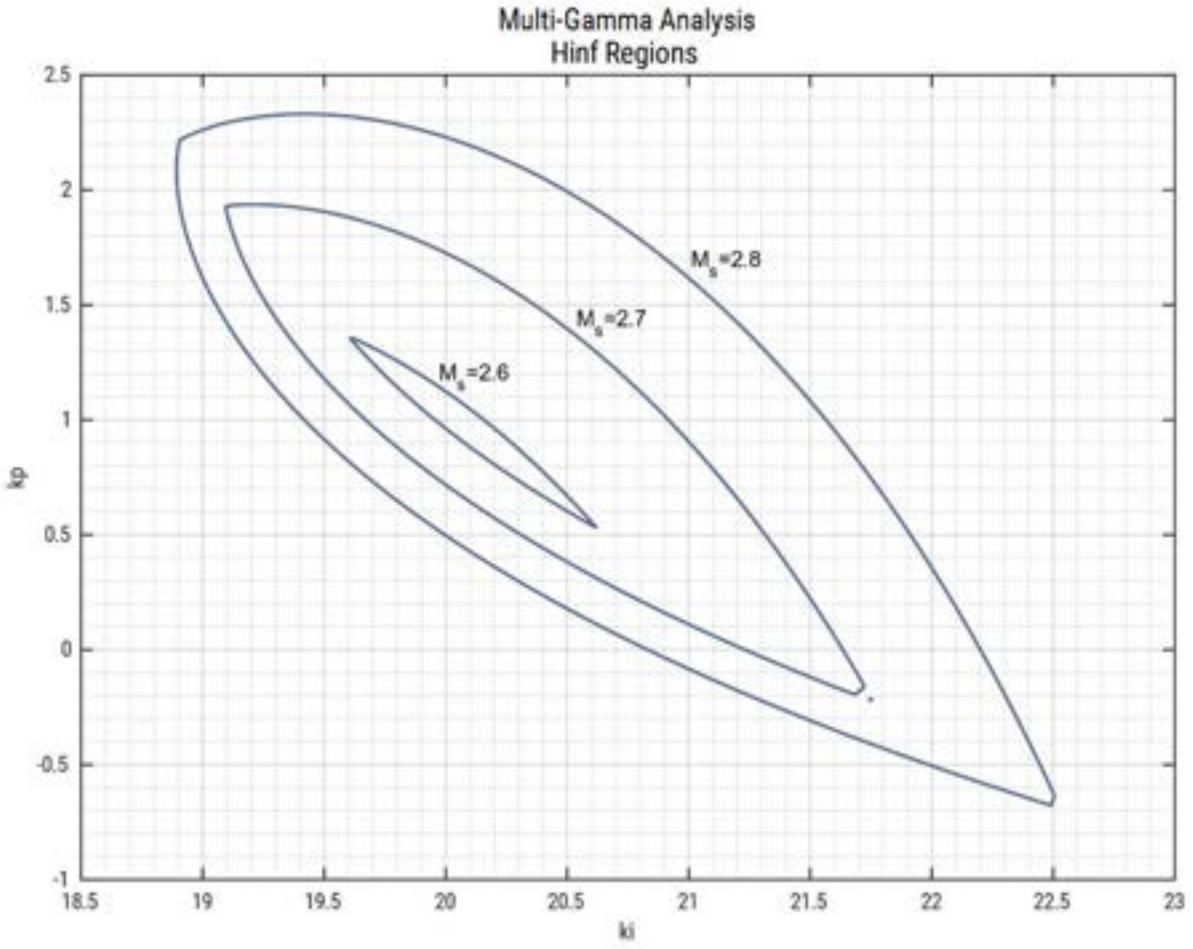
Example : H_∞ – region for unstable process:

$$P(s) = \frac{s^3 + 4s^2 - s + 1}{s^5 + 2s^4 + 32s^3 + 14s^2 - 4s + 50},$$

$$C(s,k) = k_p + \frac{k_i}{s}$$

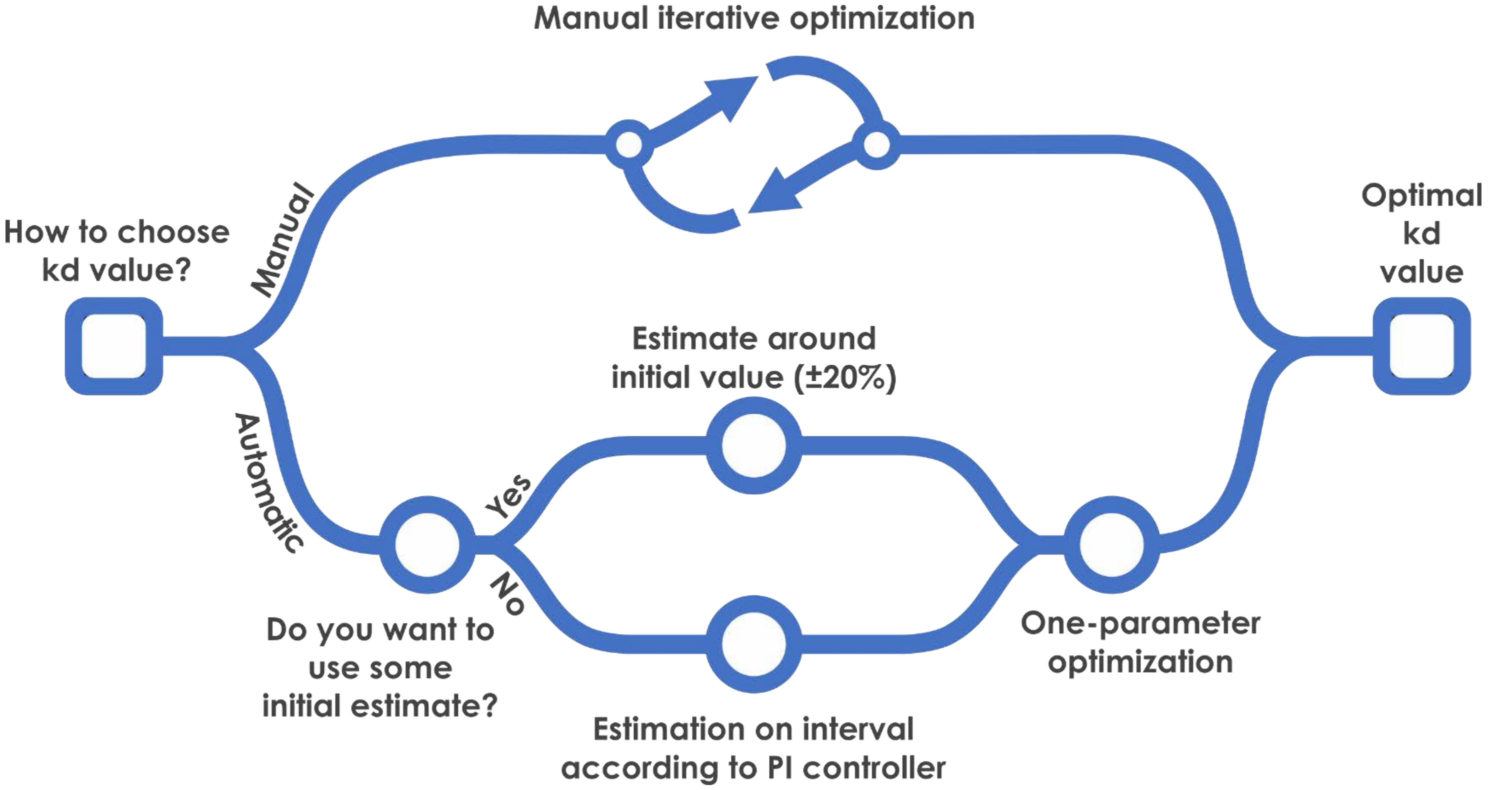
$$\left\|S(s,k)\right\|_\infty = \left\|\frac{1}{1 + C(s,k)P(s)}\right\|_\infty \le M_S$$

$$M_S \in \{2.6, 2.7, 2.8\}$$



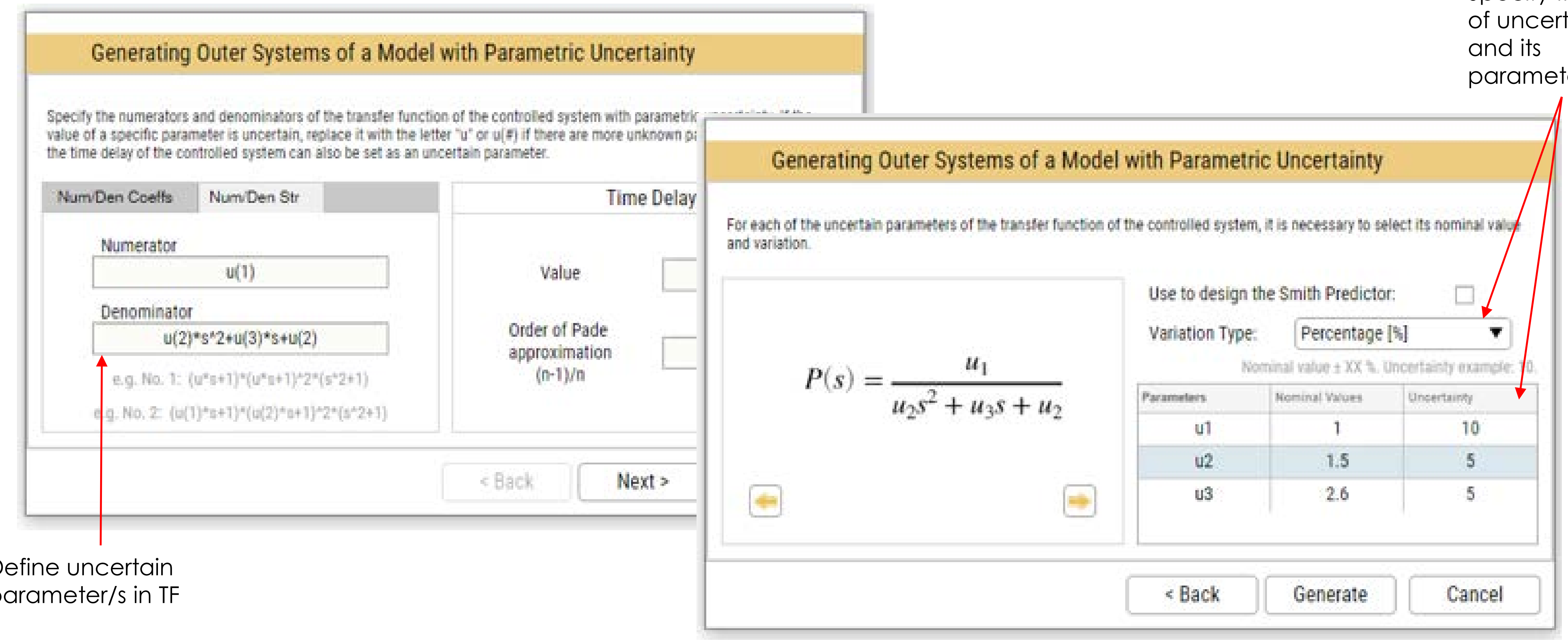
Appendix B: Selection of k_d and τ

It is recommended to start with the ideal PID controller $(\tau = 0)$. If there exists a PI controller for the given design specification with parameters k_p, k_i, $(T_i = k_p / k_i)$, then it is recommended to estimate optimal k_d in the interval $\left[0.2k_p^2/k_i, 0.3k_p^2/k_i\right]$ manually or with the help of GUI build-in function .



Appendix C: PID H$_\infty$ Designer GUI – Parametric Uncertainty (P.U.)

Specify the type of uncertainty and its parameters



Define uncertain parameter/s in TF

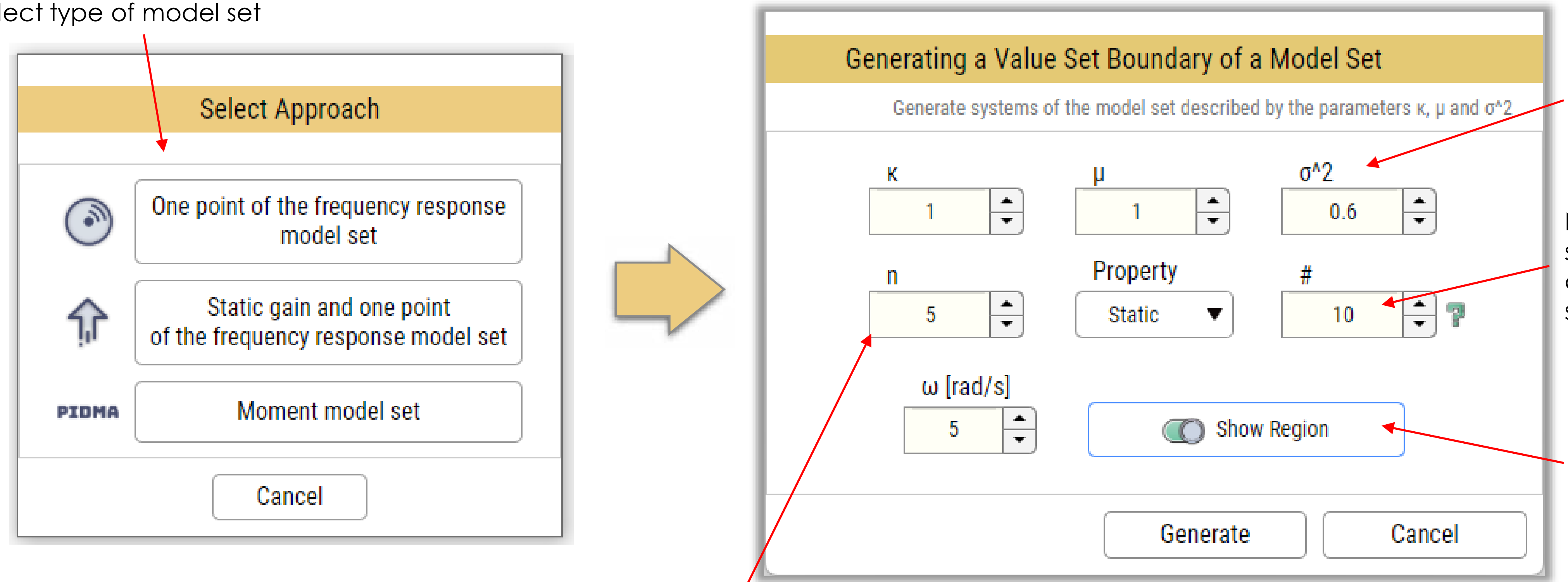
Appendix D: PID H_∞ Designer GUI – Experimental Model Set (E.M.S.)

Select type of model set

Model set parameters

Number of systems on the curve of value set of model set

Plot the value set at frequency omega



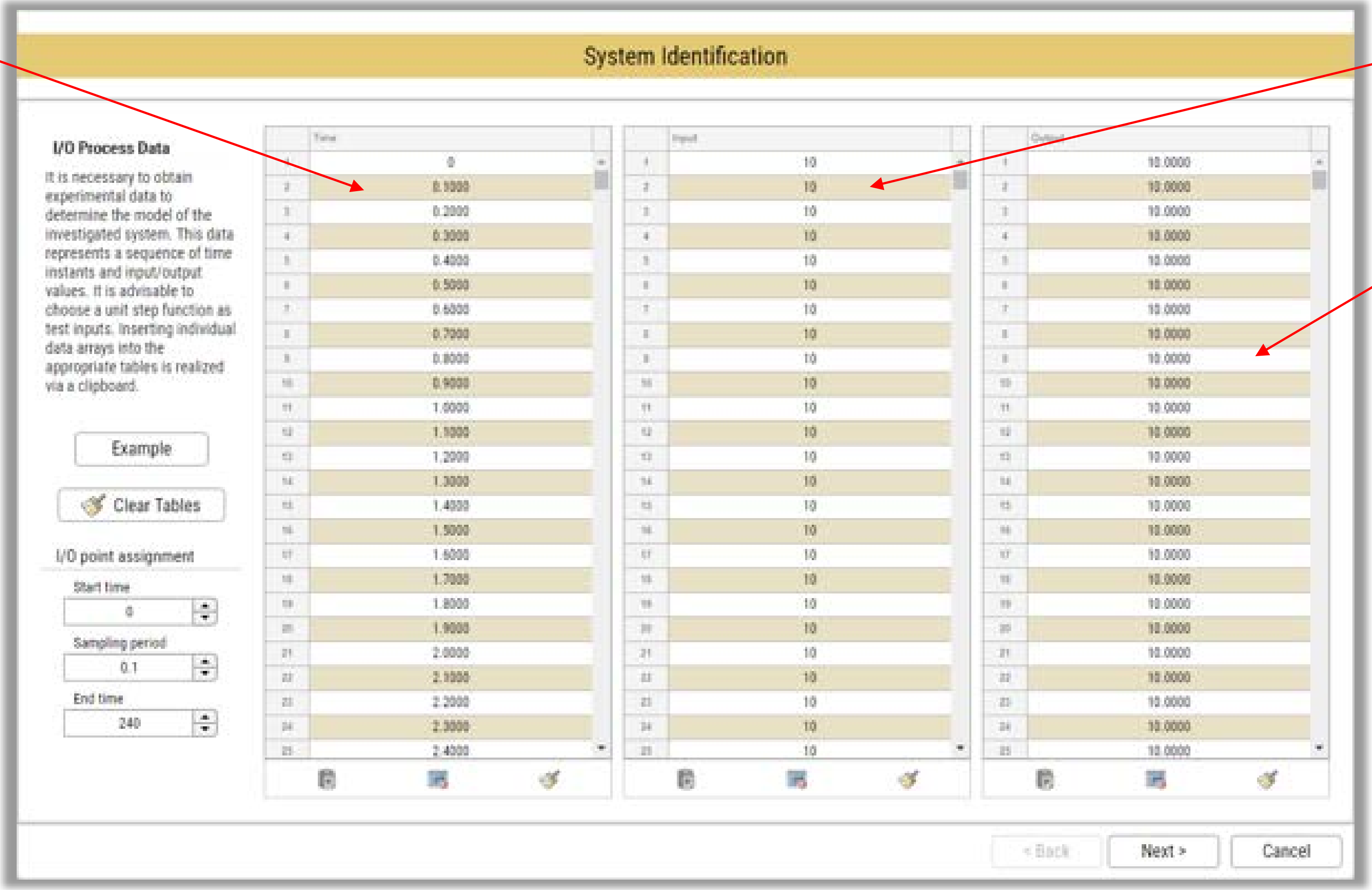
Maximal order of system

Appendix E: PID H_∞ Designer GUI –

System Identification (1)

Time samples

A user must use the clipboard function which is integrated into the window manager to enter Input/Output data. This tool is accessible through the control panel on the left side of the screen.

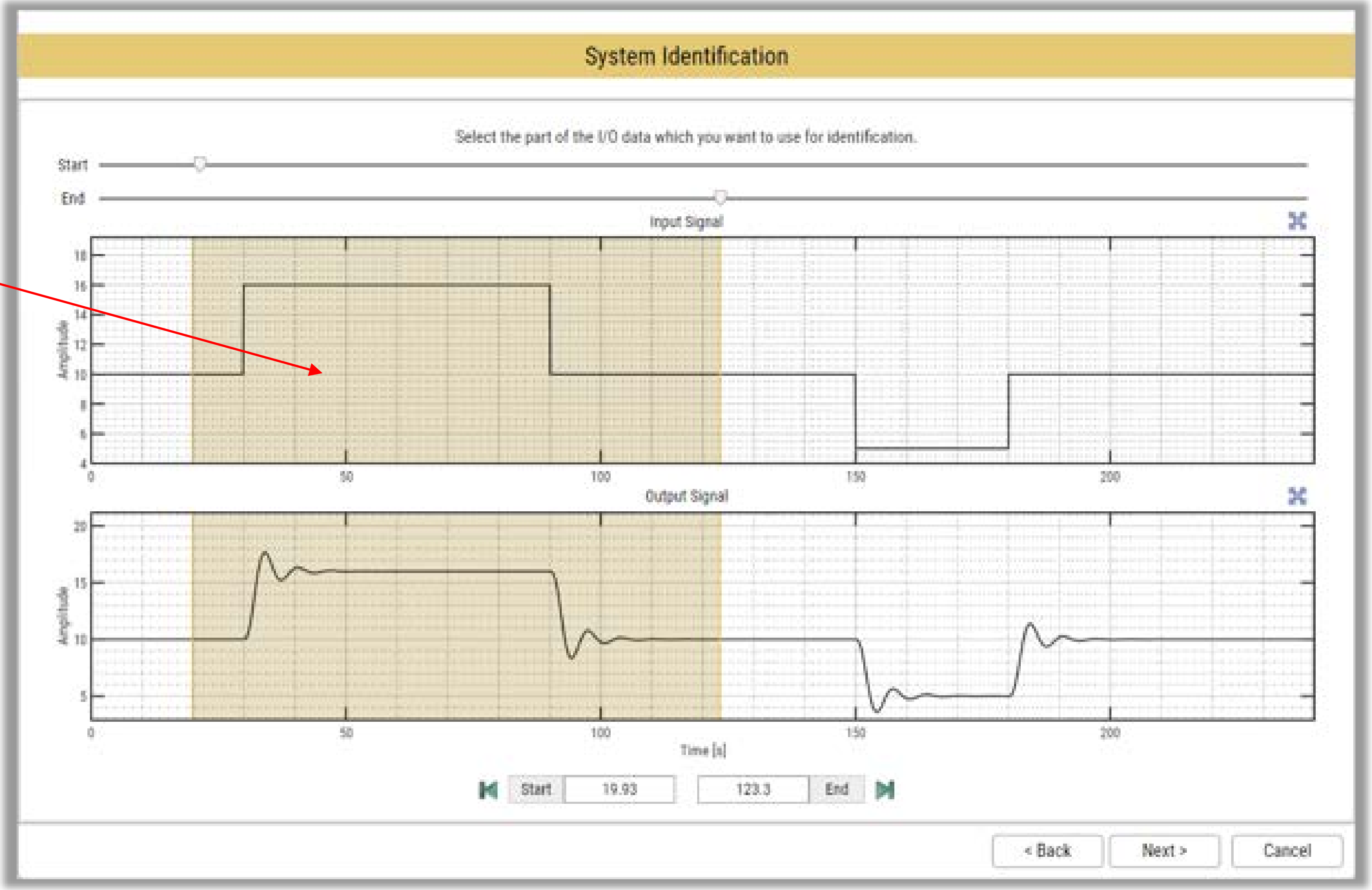


Input data samples

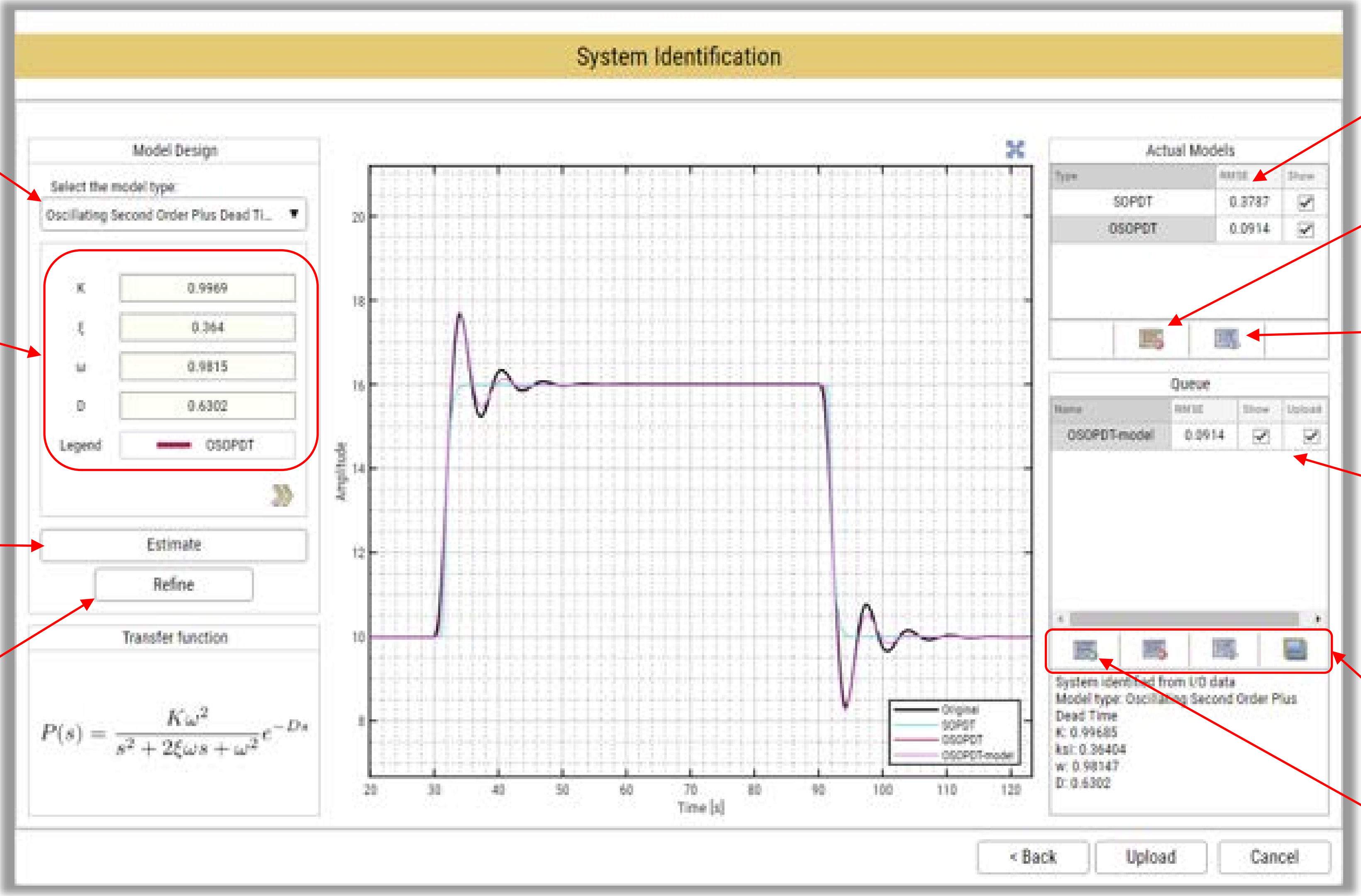
System output samples

Appendix E: PID H$_\infty$ Designer GUI – System Identification (2)

Select part of I/O data to identification



Appendix E: PID H_∞ Designer GUI – System Identification (3)



Type of identified model

Identified parameters of model

Run identification

Run additional numerical fitting

Identified model types

Delete selected actual model/s

Hide selected actual model/s response/s

Queue of models for importing to the System Editor

Queue models editing

Add selected/new model/s to the queue

Appendix F: Application Examples

Magnetic Levitation System

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{F_{em1}}{m} + \frac{F_{em2}}{m} + g$$

$$\dot{x}_3 = \frac{1}{f_i(x_1)}\left(k_i u_1 + c_i - x_3\right)$$

$$\dot{x}_4 = \frac{1}{f_i(x_d - x_1)}\left(k_i u_2 + c_i - x_4\right)$$

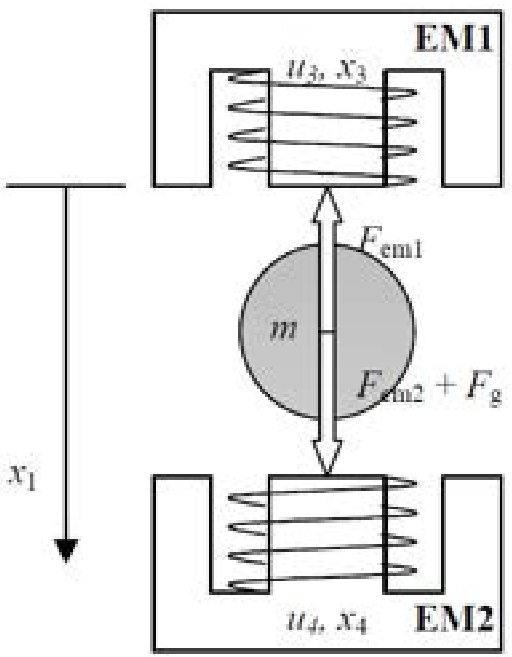
where

$$F_{em1} = x_3^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_1}{F_{emP2}}}$$

$$F_{em2} = x_4^2 \frac{F_{emP1}}{F_{emP2}} e^{-\frac{x_d - x_1}{F_{emP2}}}$$

$$f_i(x) = \frac{f_{iP1}}{f_{iP2}} e^{-\frac{x}{f_{iP2}}}$$

F_{em1}—attraction force of the upper electromagnet [N],
F_{em2}—attraction force of the lower electromagnet [N],
F_g—force of gravity [N],
g—acceleration of gravity—9.81 [m/s²]
m—mass of ball—0.0571 [kg],
u_1—electric voltage of the upper coil—<u_{min}, 1>,
u_{min} = 0.00498 [V],
u_2—electric voltage of the lower coil—<u_{min}, 1> [V],
x_d—distance between the magnets minus the ball diameter—defined by user [m],
x_1—distance from the upper magnet to ball —<0, 0.016> [m],
x_2—linear speed of the ball [m/s]
x_3—coil current of the upper electromagnet —<i_{min}, 2.38>,
i_{min} = 0.03884 [A],
x_4—coil current of the lower electromagnet —<i_{min}, 2.38> [A].



$c_i = 0.0242$ [A]
$F_{emP1} = 1.7521 \times 10^{-2}$ [H]
$F_{emP2} = 5.8231 \times 10^{-2}$ [H]
$f_{iP1} = 1.4142 \times 10^{-4}$ [ms]
$f_{iP2} = 4.5626 \times 10^{-3}$ [m]
$k_i = 2.5165$ [A]

Magnetic Levitation System: Linear Model Set

Transfer Functions from u1 to x1 (u2=0)

$$P_1(s) = \frac{-2.0893e4}{s^3 + 186.2891 \cdot s^2 - 1.6847e3 \cdot s - 3.1384e5}, \qquad (x_1 = 8[\mathrm{mm}])$$

$$P_2(s) = \frac{-2.7277e4}{s^3 + 288.7746 \cdot s^2 - 1.6847e3 \cdot s - 4.8649e5}, \qquad (x_1 = 10[\mathrm{mm}])$$

$$P_3(s) = \frac{-3.5611e4}{s^3 + 447.6417 \cdot s^2 - 1.6847e3 \cdot s - 7.5413e5}, \qquad (x_1 = 12[\mathrm{mm}])$$

[ML1] Hypiusová M., Kozáková A.: *Robust PID Controller Design for the Magnetric Levitation System: Frequency Domain Approach*. 21st International Conference on Process Control (PC), June 6-9, 2017, Štrbské Pleso, Slovakia

PID H_∞ Designer

Input :

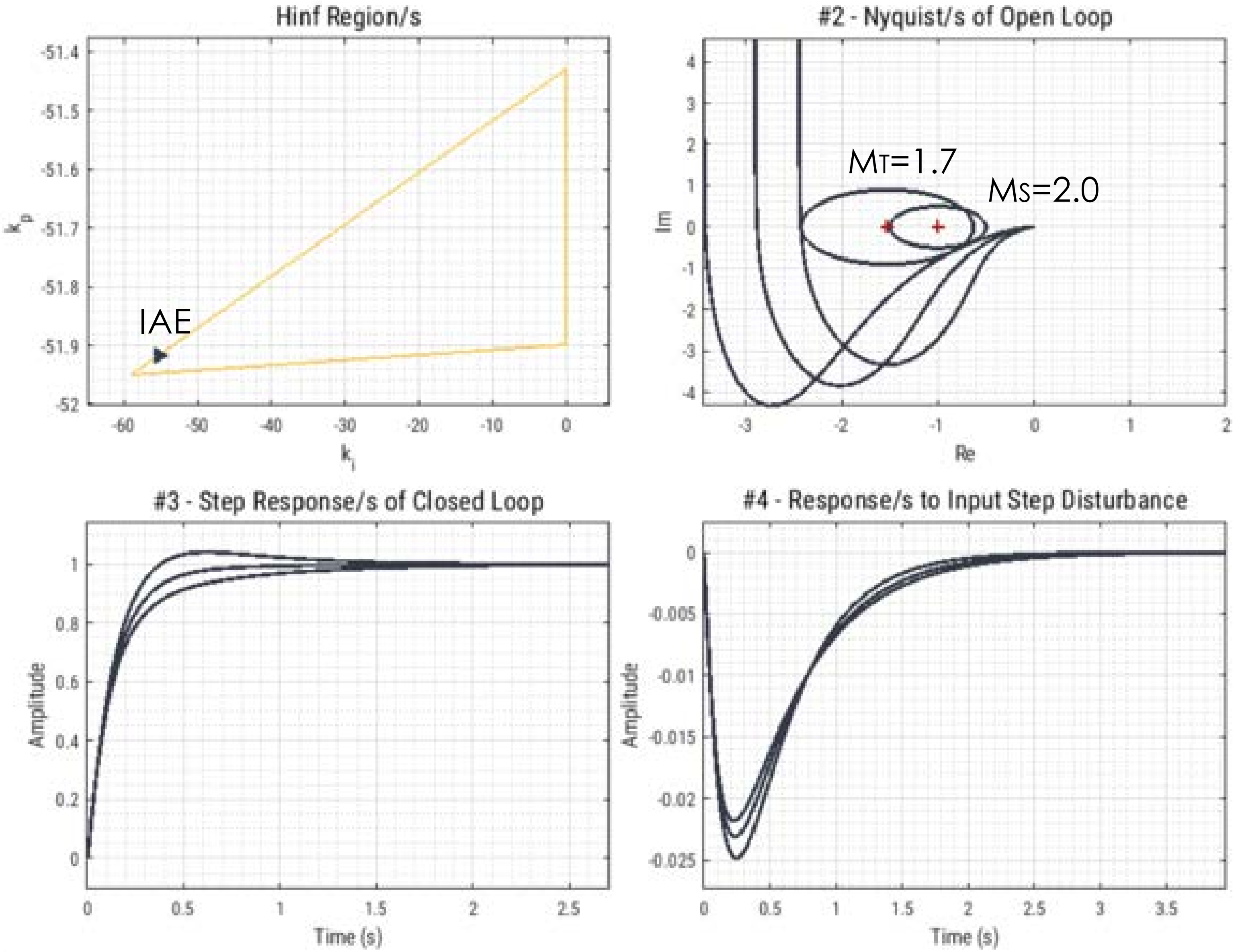
Model Set: $\{P_1, P_2, P_3\}$

Design specification:

2DOF PID controller

Setpoint tracking, IAE

$M_S \leq 2.0, \quad M_T \leq 1.7$



Output :

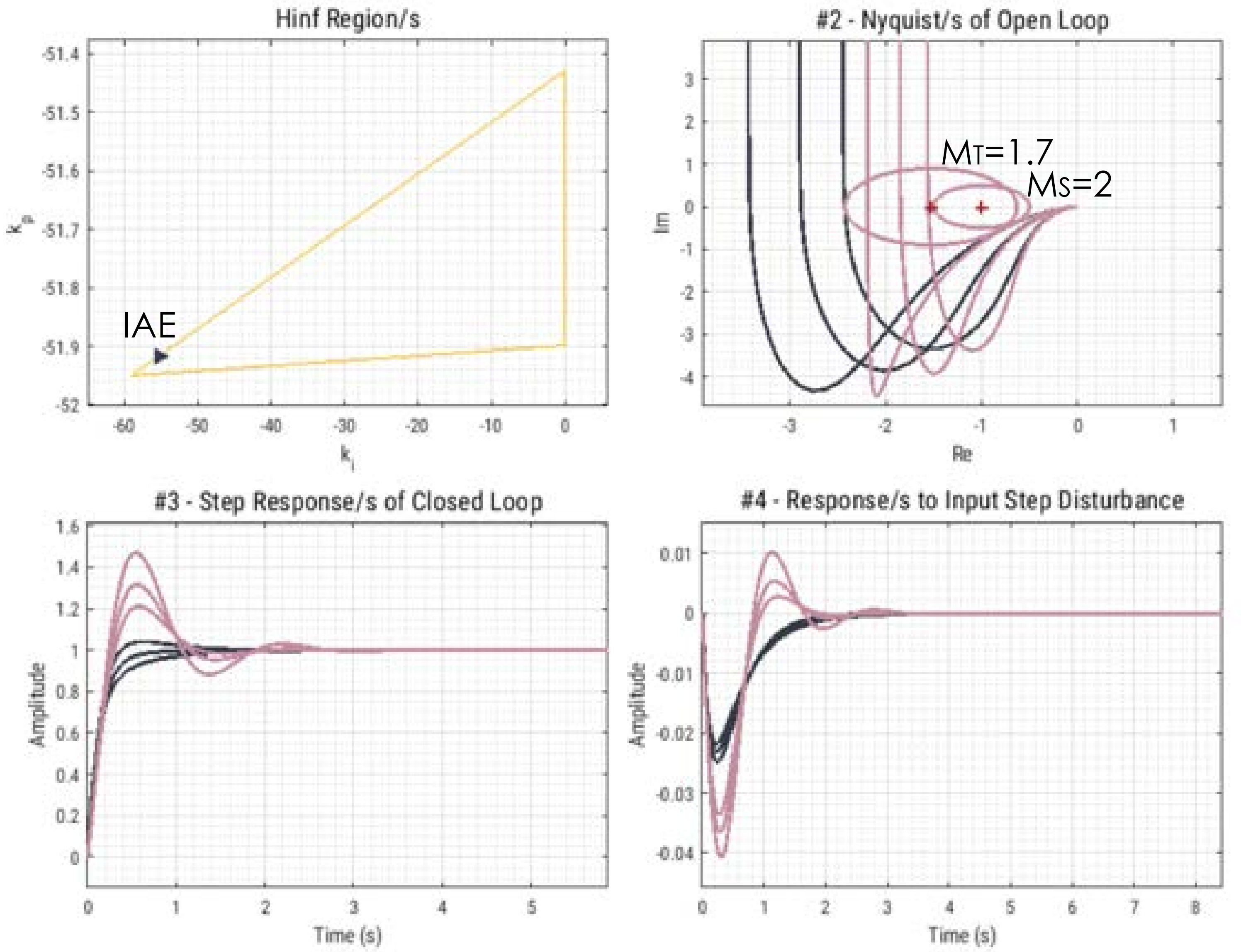
$k_p = -51.95$

$k_i = -59.07$

$k_d = -3.63$

$b = 0.5, \ c = 0.0$

Comparison with the PID-controller proposed in [ML1]



PID H$_\infty$ Designer:

2DOF PID – controller

$k_p = -51.95$

$k_i = -59.07$

$k_d = -3.63$

$b = 0.5, \ c = 0.0$

[ML1]:

1DOF PID – controller

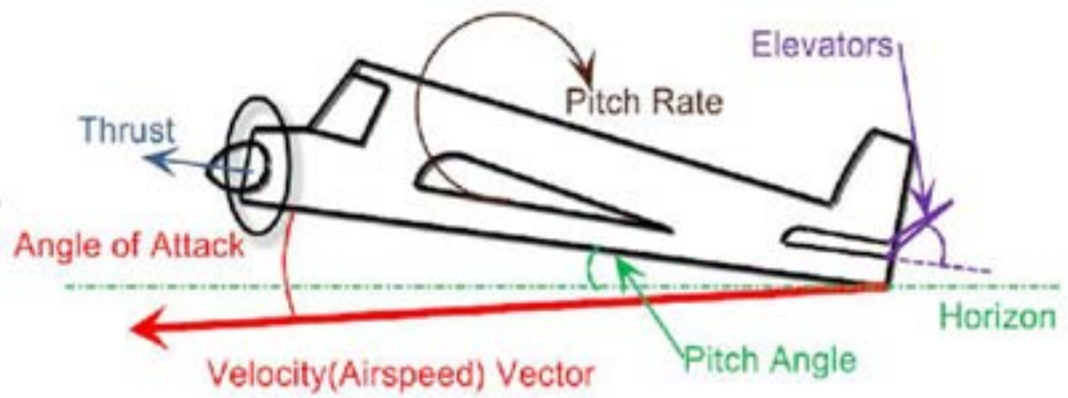
$k_p = -33.27$

$k_i = -61.04$

$k_d = -4.532$

Longitudinal motion of F4E fighter aircraft

We consider a model of the longitudinal motion of an F4E fighter aircraft [LM1], [LM2]. The input is the elevator position, the output is the pitch rate, and the system is linearized around four representative flight conditions:



$$P_i(s) \triangleq \frac{b^i(s)}{a^i(s)}, \quad i = 1,\ldots 4.$$

Mach 0.5, 5000 ft: $a^1(s) = -52.75 + 22.00s + 15.84s^2 + s^3, \quad b^1(s) = -163.8 - 185.4s$

Mach 0.85, 5000 ft: $a^2(s) = -122.5 + 34.93s + 17.12s^2 + s^3, \quad b^2(s) = -789.1 - 507.8s$

Mach 0.9, 35000 ft: $a^3(s) = -14.64 + 17.51s + 15.33s^2 + s^3, \quad b^3(s) = -101.8 - 158.3s$

Mach 1.5, 35000 ft: $a^2(s) = 269.1 + 43.60s + 15.74s^2 + s^3, \quad b^4(s) = -251.4 - 304.2s$

[LM1] J. Ackermann. *Robust Control Systems with Uncertain Physical Parameters*. Springer Verlag, Berlin, 1993.
[LM2] Henrion D.,Šebek M.,Kučera V.: *Positive polynomials and robust stabilization with fixed - order controllers.* IEEE Trans. Automatic Control AC-48 (2003),7.

PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PI controller

Setpoint tracking, IAE

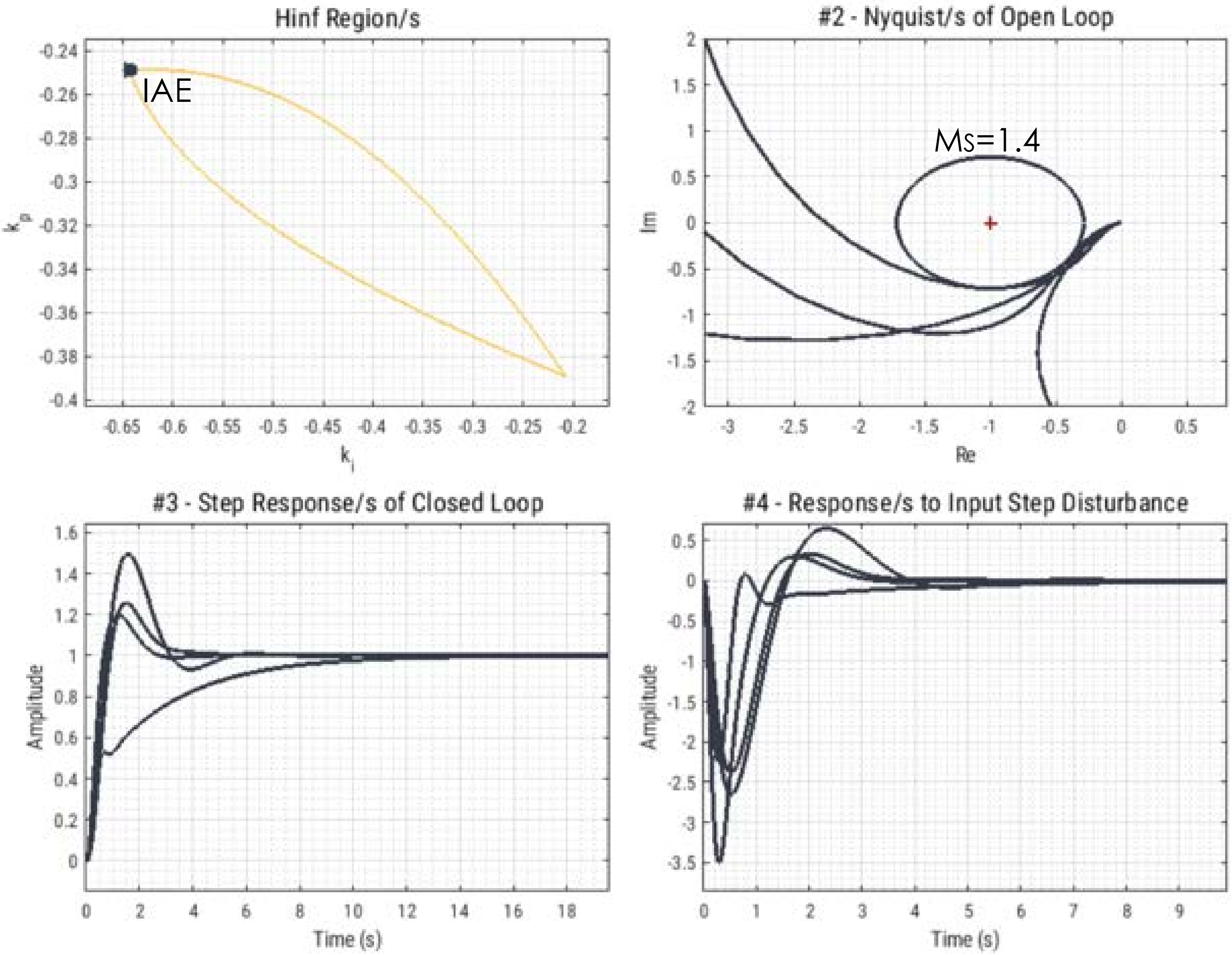
$M_S \leq 1.4$

Output :

$k_p = -0.25$

$k_i = -0.64$

$b = 0.0, \;\; c = 0.0$



PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3, P_4\}$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$M_S \leq 1.4, \quad M_T \leq 1.4$

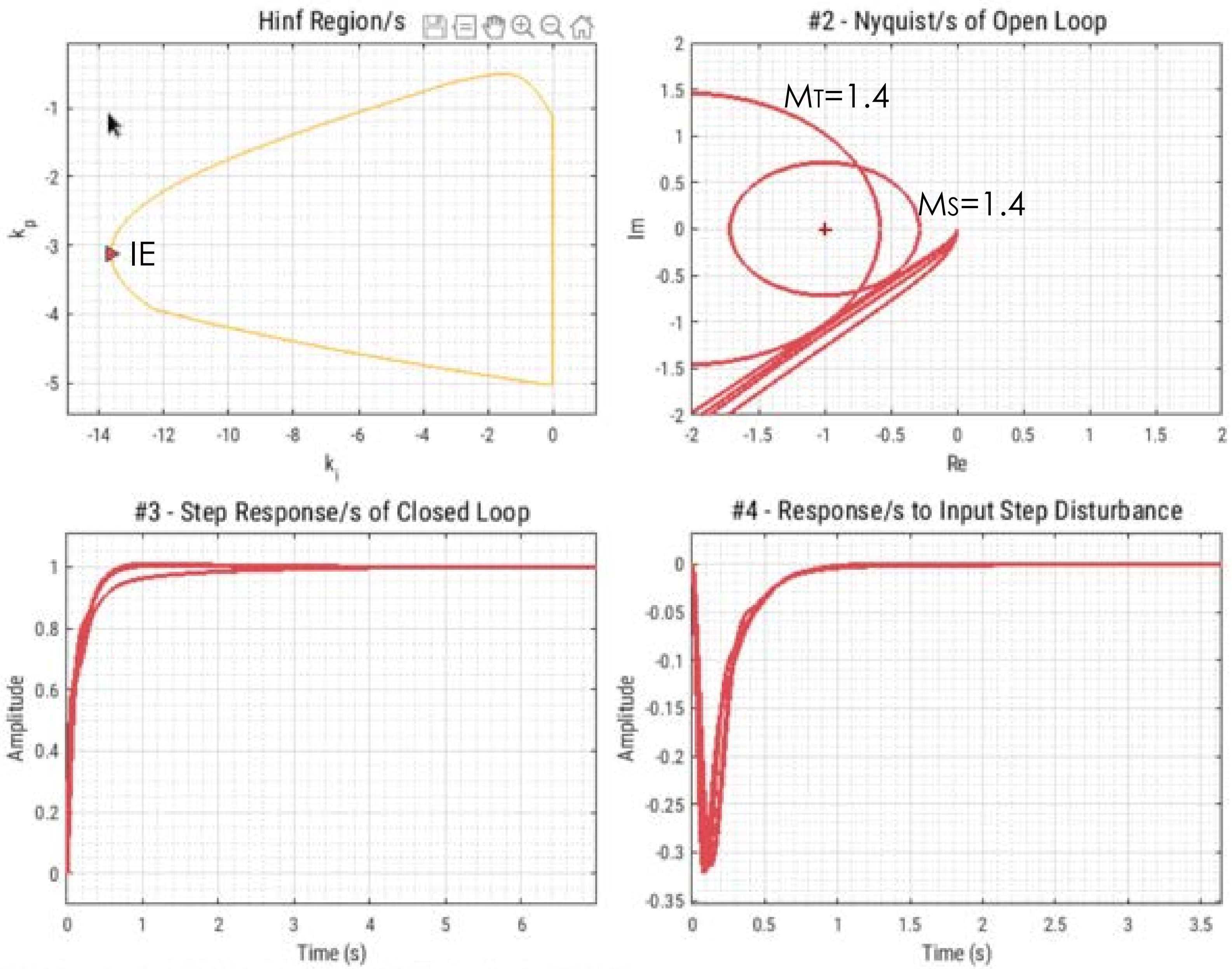
Output :

$k_p = -3.12$

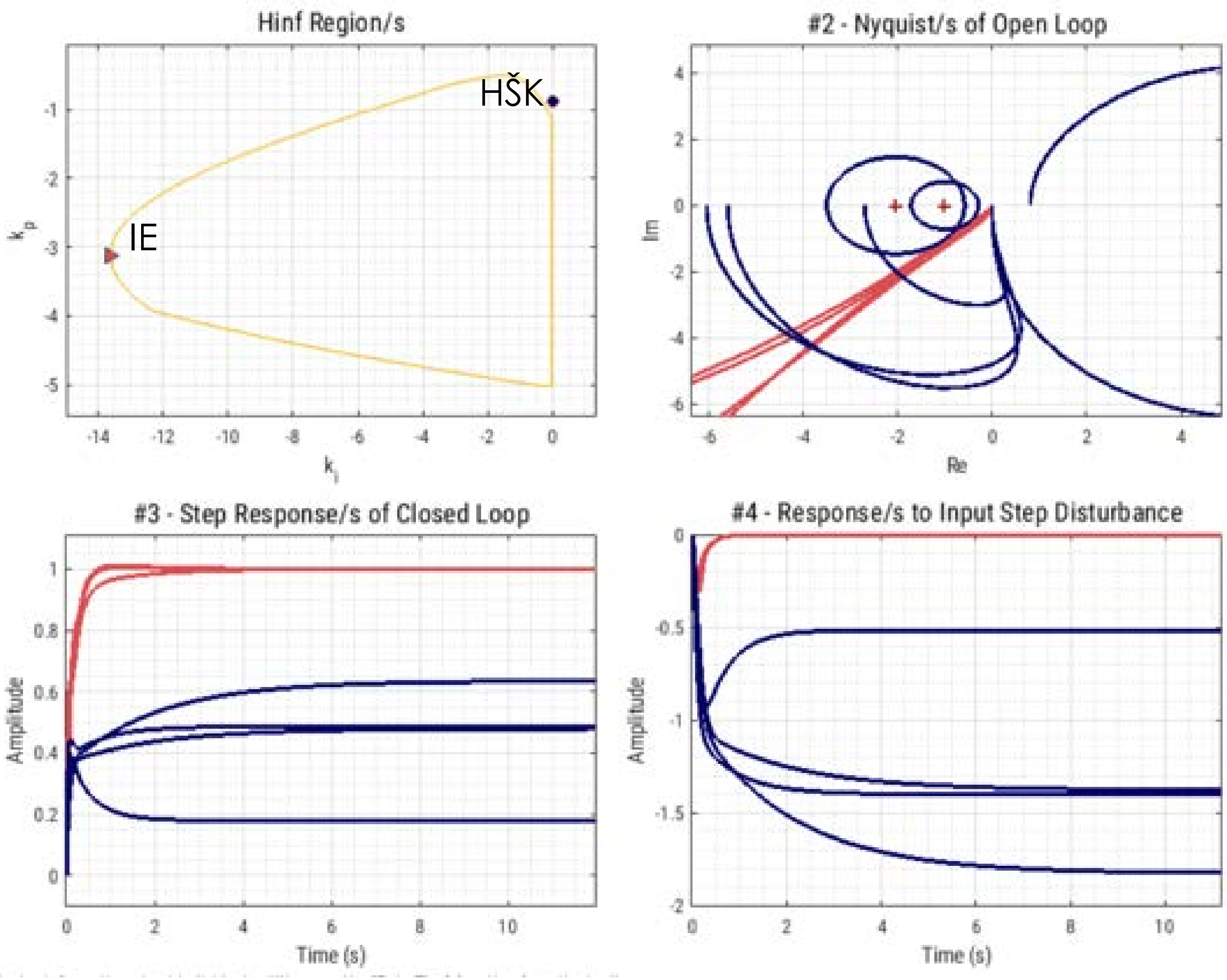
$k_i = -13.63$

$k_d = -0.06$

$b = 0.4, \;\; c = 0.6$



Comparison with the P-controller proposed in [LM2]



PID H$_\infty$ Designer:

2DOF PID – controller

$k_p = -3.12$

$k_i = -13.63$

$k_d = -0.06$

$b = 0.4, \ c = 0.6$

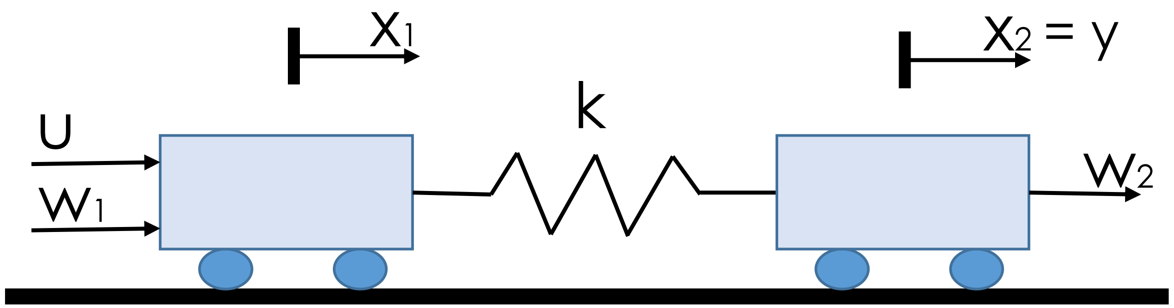
HŠK [LM2]:

P – controller

$k_p = -0.8698$

Benchmark Problem for Robust Control

Wie, B. and D.S. Bernstein (1990). A benchmark problem for robust control design. In: *Proc. American Control Conference*. San Diego, CA, USA. pp. 961–962.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix}=\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -k/m_1 & k/m_1 & 0 & 0\\ k/m_2 & -k/m_2 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1/m_1\\ 0\end{bmatrix}u+\begin{bmatrix}0 & 0\\ 0 & 0\\ 1/m_1 & 0\\ 0 & 1/m_2\end{bmatrix}\begin{bmatrix}w_1\\ w_2\end{bmatrix}$$

$$P_{ux_2}(s,k)=\left\{\frac{k}{s^2(s^2+2k)}:k\in[0.5,2]\right\}\rightarrow P(s,k,\xi)=\frac{k}{s\left(s^2+2\xi\sqrt{2k}\cdot s+2k\right)},\quad \xi\rightarrow 0$$

$$\uparrow \qquad\qquad\qquad\qquad \downarrow$$

$$PD\ ({}^2k_p={}^1k_i,\ {}^2k_d={}^1k_p)\quad\leftarrow\quad PI\ ({}^1k_p,\ {}^1k_i)$$

PID H_∞ Designer

Input :

Model Set: $\{P_1, P_2, P_3\}$

$P_1(s) = P(s, 0.5, 0.1),$

$P_2(s) = P(s, 1.0, 0.1),$

$P_3(s) = P(s, 2.0, 0.1).$

Design specification:

1DOF PI + compensator $F(s)$

$$F(s) = \left(\frac{\Omega^2}{\left(s^2 + 2\xi\,\Omega\, s + \Omega^2\right)} \right)^2$$

$\Omega = 0.9, \;\; \xi = 0.7$

Setpoint tracking, IE

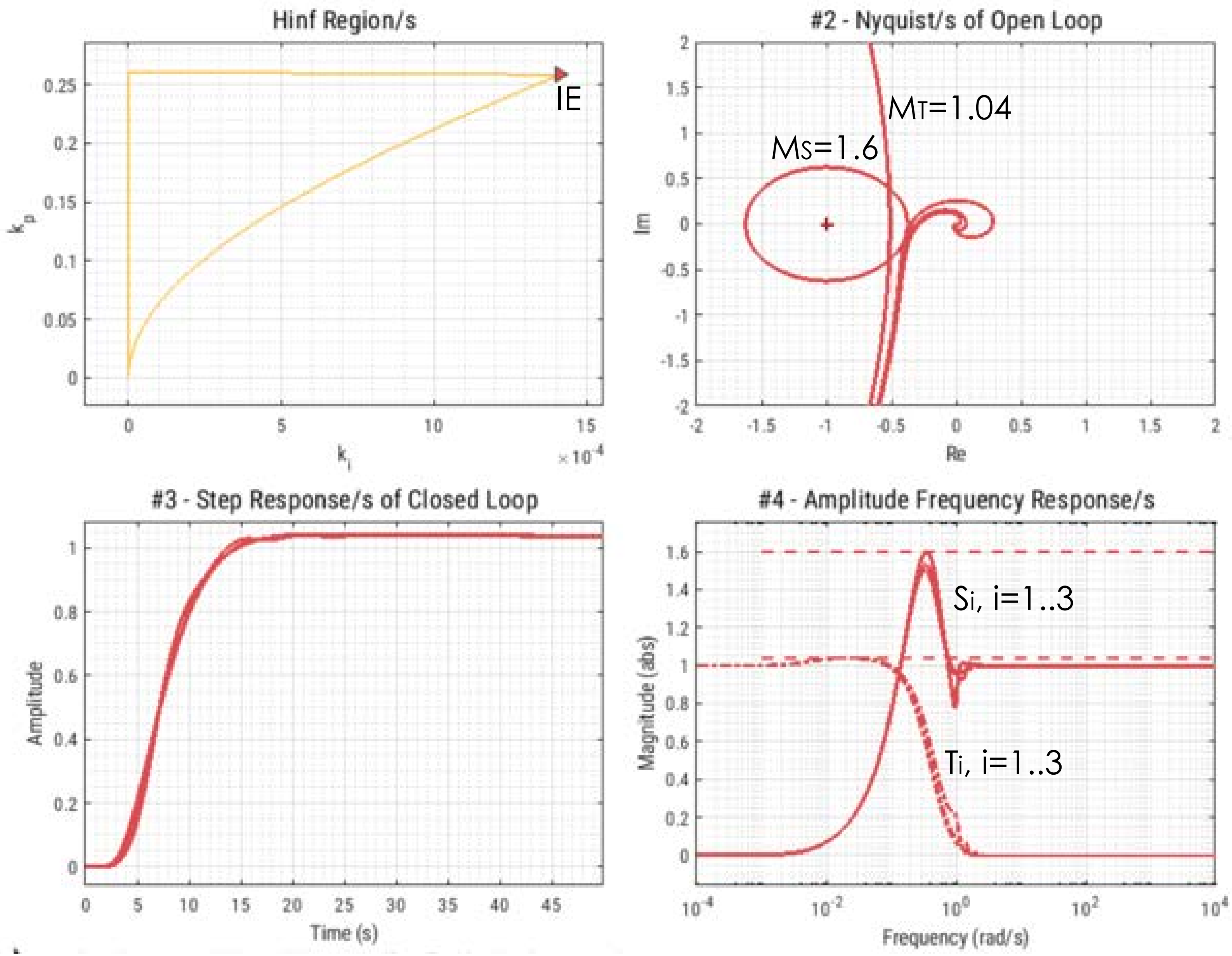
$M_S \leq 1.4, \;\; M_T \leq 1.05$

Output :

$k_p = 0.2586$

$k_i = 0.001413$

$b = 0.0$



PID Controller Design using One Frequency Point

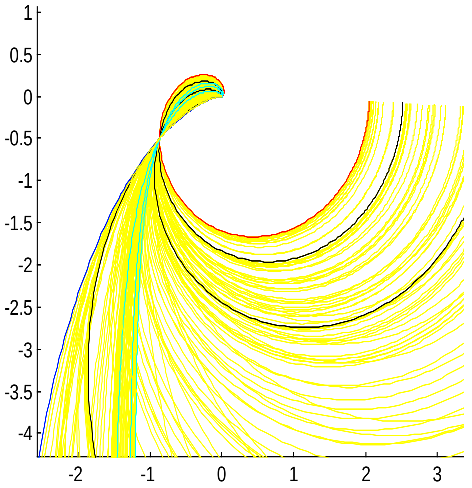
- SCHLEGEL, M.: *Nový přístup k robustnímu návrhu průmyslových regulátorů*. Habilitační práce, Západočeská univerzita v Plzni, 2000. https://www.schlegel.zcu.cz/downloads.php?lng=eng
- SCHLEGEL, M.: *Exact Revision of the Ziegler-Nichols Frequency Response Method*. In Proceedings of the IASTED International Conference Control and Application, Cancun, Mexico, 2002, p. 121-126. ISBN 088986330X, ISSN 1025-8973 .

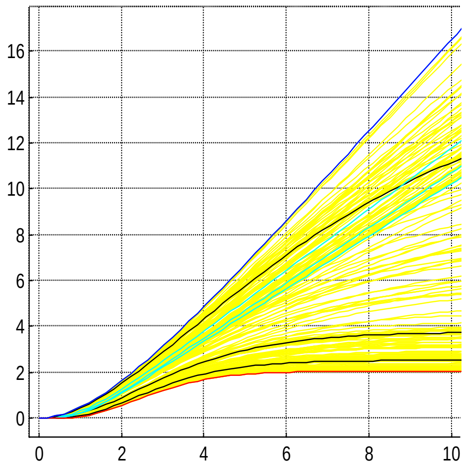
Definition (One Point Model Set). We are given one disturbance free sample of the plant frequency responce $_1\ \omega_1$ and a fixed $\in\{2 \quad \infty\}$. A plant model is an element of the plant family $^{n}_{\mathbb{R}^-}\left(\ _1\ \omega_1\right)$ if it is consistent with the two following conditions:

(i) (A priori Hyposisis)

$$P\left(s\right)=\frac{1}{p(s)},$$

where $p(s),\ \deg(p(s))\leq n,$ is a polynomial with real nonnegative coeficients, and all roots of $p(s)$ lie in the interval $\left(-\infty,0\right]$.

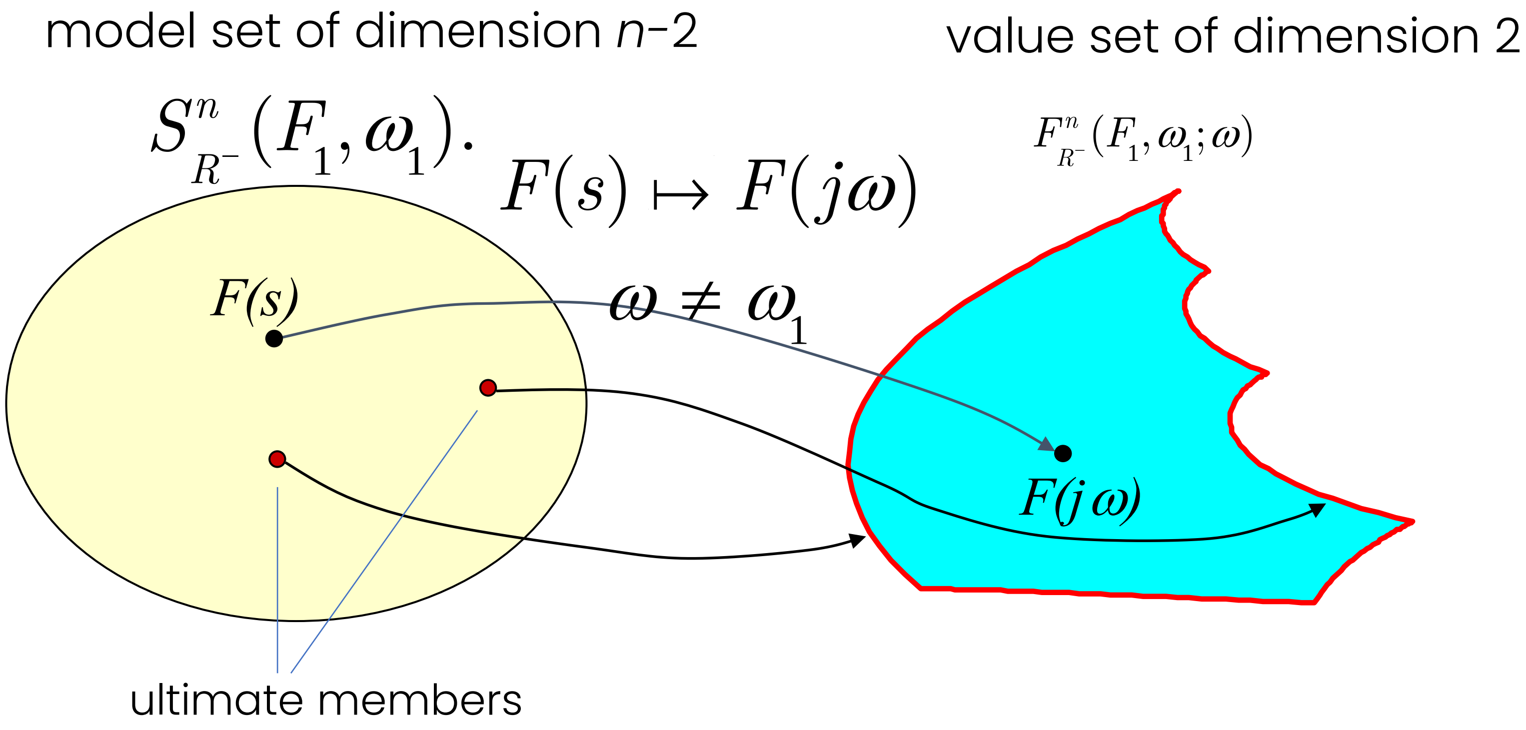




(ii) (Experimental Data Interpolation)

$$P(j\omega_1)=F_1,\quad -2\pi<\arg P(j\omega_1)\leq 0.$$

Main Idea of Solution



Only ultimate members of the unfalsified plant family can play an active role in the Nyquist curve constraints.

PID H_∞ Designer

Input :

Model Set:

$S^{n}_{\mathbb{R}^-}(F_1, \omega_1)$,

$n = 10,\ F_1 = e^{-1.8j},\ \omega_1 = 1$

Design specification:

2DOF PI controller

Setpoint tracking, IE

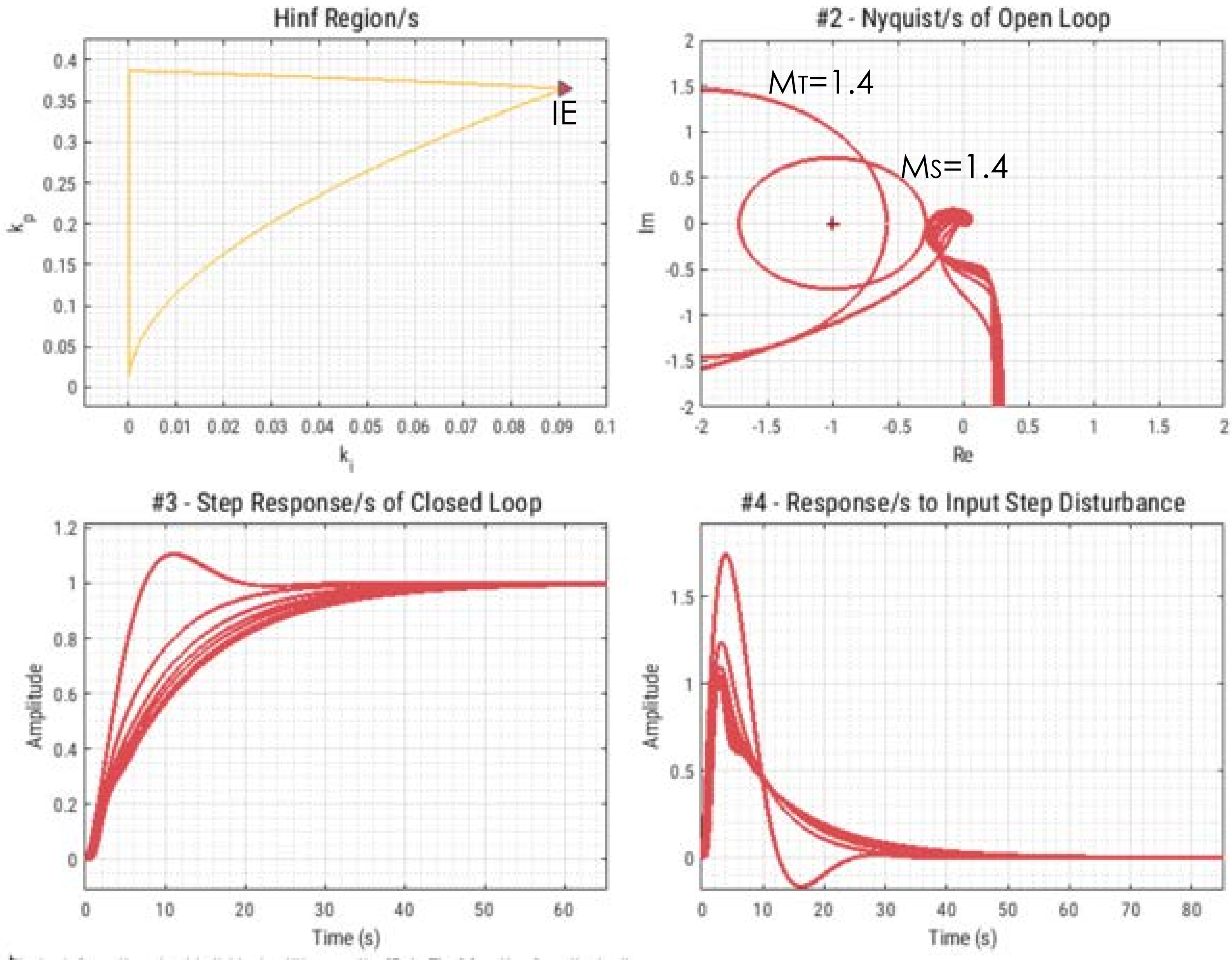
$M_S \leq 1.4, M_T \leq 1.4$

Output :

$k_p = 0.37$

$k_i = 0.091$

$b = 0.3$



PID-Autotuner PIDMA

- SCHLEGEL M.: *Nový přístup k robustnímu návrhu průmyslových regulátorů*. Habilitační práce, Západočeská univerzita v Plzni, 2000. https://www.schlegel.zcu.cz/downloads.php?lng=eng
- SCHLEGEL M., Večerek O.: *Robust design of Smith predictive controller for moment model set* . Proceedings of the 16th IFAC World Congress, p. 427-432, Elsevier, Oxford, 2006.
- SCHLEGEL M., BALDA P., ŠTĚTINA M.. *Robustní PID autotuner: momentová metoda*. Automatizace, 46(4):242–246, 2003.

Definition ((κ,μ,σ^2) - Model Set). We are given the first three moments m_0, m_1, m_2 of the process impulse response $(\ )$ and fixed $\in\{2,..,\infty\}$. A transfer function $(\ )$ is an element of the plant family $^{n}_{\mathbb{R}^-}(\kappa,\mu,\sigma^2)$ if it is consistent with the two following conditions:

(i) (A priori Hyposisis)

$$(\ ) = \frac{1}{p(s)}$$

where $(\ )$, $\deg(\ (\ )) \le$, is a polynomial with real nonnegative coeficients, and all roots of $p(s)$ lie in the interval $(-\infty, 0]$.

(ii) (Experimental Data)

$$_i = \int_0^\infty \ ^i \qquad = 0\ 1\ 2,$$

$$\kappa = \ _0,$$

$$\mu = m_1/m_0,$$

$$\sigma^2 = m_2/m_1 - m_1^2/m_0^2.$$

PID H$_\infty$ Designer

Input :

Model Set:

$$S^{n}_{\mathbb{R}^-}\left(\kappa,\mu,\sigma^2\right),$$

$$n = 20,\ \kappa = 1,\ \mu = 1,\ \sigma^2 = 0.6$$

Design specification:

2DOF PID controller

Setpoint tracking, IE

$$M_S \le 1.6, M_T \le 1.1$$

Output :

$$k_p = 2.587$$

$$k_i = 4.311$$

$$k_d = 0.25$$

$$b = 0.8,\ \ c = 1$$