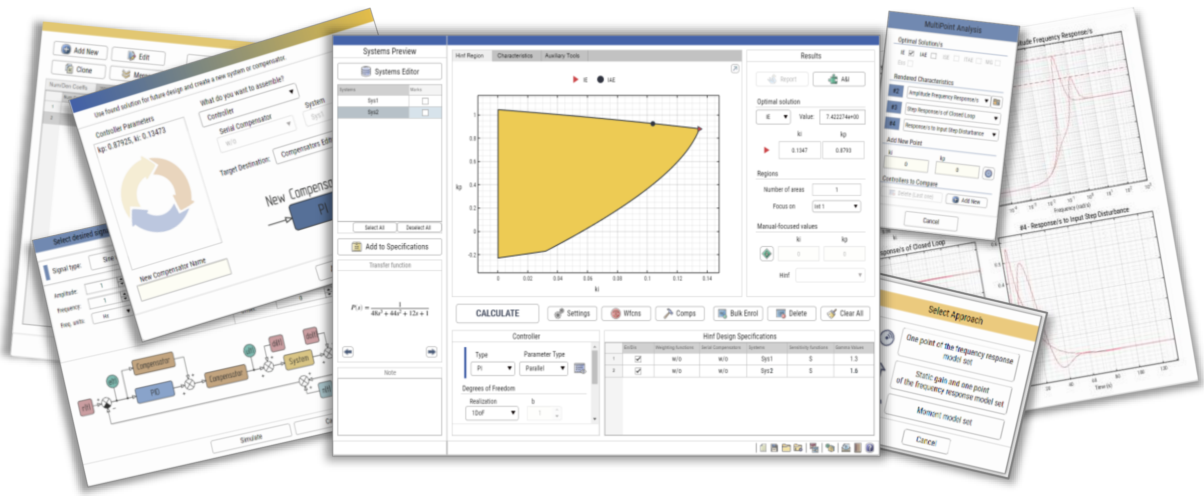
PID H$_\infty$ Designer

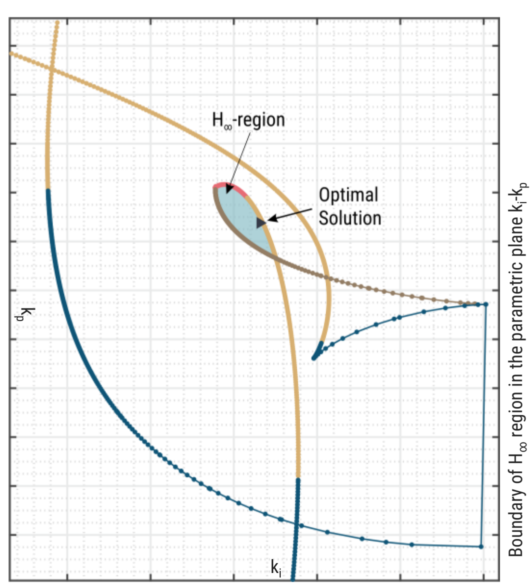
Tool for Analytical Design of a Wide Class of Practically Applicable Controllers

PID H$_\infty$ Designer is the first advanced, easy-to-use web-design tool for the analysis and analytical design of a wide class of controllers, including PI, PD, PID, PR (proportional-resonant), Lead-Lag, RC (repetitive control), SP (Smith predictor), and many more industrial controllers, all based on H$_\infty$ performance and robustness constraints.



General Basic Problem Formulation

The basic supported design problem considers a simple control loop with an LTI-SISO system and searches for all controllers of a given structure with two tunable parameters guaranteeing the internal stability of the loop and satisfying one given H$_\infty$ requirement in the form of an inequality. Each compliant controller is represented by one point in the parametric plane of the controller. Points satisfying the design requirement form the so-called H$_\infty$ region. The analytical solution to the basic design task now consists of finding the boundary of this H$_\infty$ region. Finally, an additional optimization criterion determines the optimal controller in this area. A typical design problem, unlike the basic one, contains a finite number of controlled system models (multi-model) and a finite number of H$_\infty$ requirements (multi-objective H$_\infty$ specification). The resulting H$_\infty$ region is then obtained as the intersection of regions relevant to individual problems. See [BS2023] for the analytical solution of the basic design problem and **www.pidlab.com** to explore the online tool.



Motivational Examples →

[BS2023] Brabec, M., Schlegel, M., Analytical Design of a Wide Class of Controllers with Two Tunable Parameters Based on H$_\infty$ Specifications, 2023, 24th International Conference on Process Control (PC), June 6-9, 2023, Štrbské Pleso, Slovakia.

Motivational Example 1

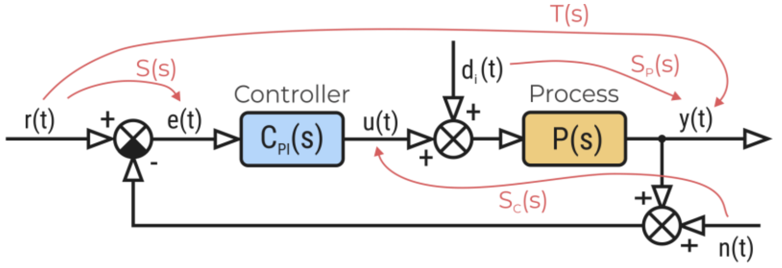
In practice, one can often meet situations where a system can be described by several transfer functions, either due to different operating conditions or changes in dynamics influenced by external or internal factors. For these reasons, it is crucial to design a controller that will meet the specified requirements under most circumstances.

Let's consider a system described by two transfer functions in two operating points

$$P_1(s) = \frac{-0.0216s + 0.0031}{s^2 + 0.457s + 0.0868} e^{-0.166s}, \qquad P_2(s) = \frac{-0.0174s + 0.0046}{s^2 + 0.5978s + 0.0445} e^{-0.166s}$$

for which we want to design a fixed **PI controller** in the form $C_{PI}(s) = K\left(1 + \frac{1}{T_i s}\right) = k_p + \frac{k_i}{s}.$

The closed-loop system is associated with four basic transfer functions: $S(s)$, $T(s)$, $S_P(s)$, and $S_C(s)$.



These functions reflect many of the interesting properties of the closed-loop system. Requirements for the control loop can be translated into requirements on the frequency response of these functions. Essentially, the designer shapes the amplitude of these functions by design constraints in the form

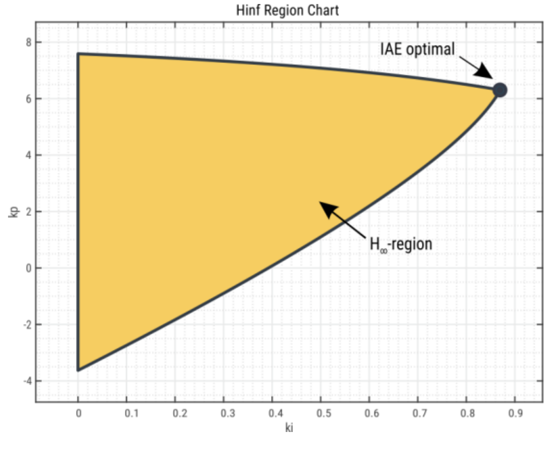
$$|S_i(j\omega)| \leq M_S, \quad \forall \omega \in \langle 0, +\infty \rangle, i = 1, 2$$

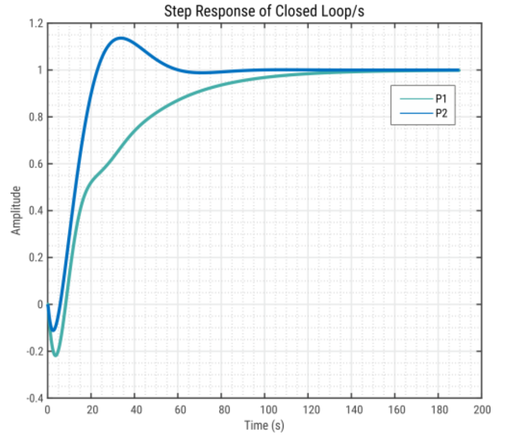
or equivalently

$$\|S_i(s)\|_\infty \leq M_S, \quad i = 1, 2$$

where $\|H\|_\infty \triangleq \max_\omega |H(j\omega)|$ is called H$_\infty$ -norm. By specifying a design constraint on the sensitivity function $S(s)$, which is a good performance indicator, in the form of maximal peak requirement can be prevented load disturbance amplification at middle frequencies and introduce a certain level of robustness. This design approach is also applicable to other transfer functions of the closed-loop system mentioned above.

By setting the maximal peak M_S to 1.6 with design focus to load step disturbance rejection (regulator problem), **PID H$_\infty$ Designer** determine the H$_\infty$ region in parametric plane $k_i - k_p$. With additional IAE optimization criterion, it is possible to obtain an optimal controller from the H$_\infty$ region with parameters $k_p = 6.3213$ and $k_i = 0.8704$.





Why Choose PID H_∞ Designer?

- **PID H∞ Designer** can be used for many process models (unstable, non-minimal phase, oscillating, time-delayed systems, systems of any order, etc.) and multi-model sets created from any number of transfer functions of such process.
- Supported design specifications reflect the essence of real control problems. Optimization of integral criteria IE, ISE, IAE, and ITAE under H_∞ constraints is supported for both load disturbance attenuation and set-point tracking problems.
- Designing a PI(D) controller with typical specifications using **PID H∞ Designer** is a routine procedure that does not require deeper knowledge of control theory from the user. For this reason, **PID H∞ Designer** is very suitable for practitioners.
- With more skills and efforts from the designer it should be possible to use **PID H∞ Designer** to design high-performance controllers or even a multi-loop scheme with several regulators and compensators, e.g., cascade control.
- **PID H∞ Designer** also supports simple process models from popular identification experiments. Specifically, two- or three-parameter models obtained from the step response of the process are supported, as well as models obtained from the relay experiment (based on the knowledge of one point of the frequency response). Moreover, the non-standard moment model set provided by the PIDMA-autotuner from **REX Controls** is also supported.

Example 2

This example represents the advanced possibilities of the tool. The target is to design a PI+PR controller (see the figure below) for the system $P(s) = 1/(s+1)^3$.

Step 1

Find the PI controller

$$C_{PI}(s) = k_p + \frac{k_i}{s}$$

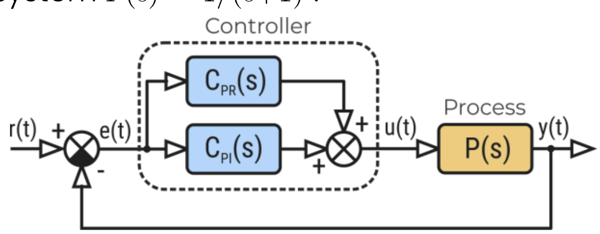
with design constraint

$$\|S(s)\|_\infty \leq 1.2$$

to track a constant reference signal.

Optimal solution (IAE)

$$k_p = 0.3554,\ k_i = 0.1714$$



Step 2

Find the PR controller parallel with the PI controller designed in step 1

$$C_{PI+PR}(s) = C_{PI} + k_q + k_r \frac{2\omega_c s}{s^2 + 2\omega_c s + \omega_0^2}$$

with design constraints

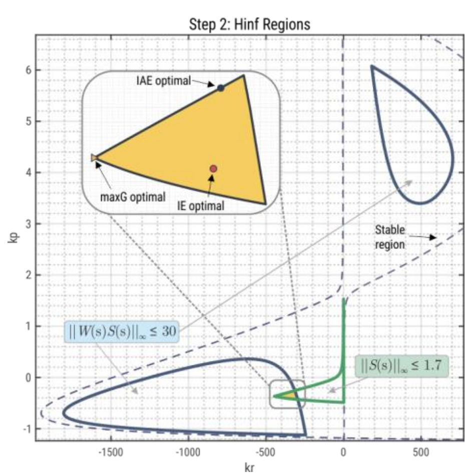
$$\|S(s)\|_\infty \leq 1.7, \qquad \|W(s)S(s)\|_\infty \leq 30$$

to attenuate a harmonic disturbance signal at the frequency 1 rad/s.

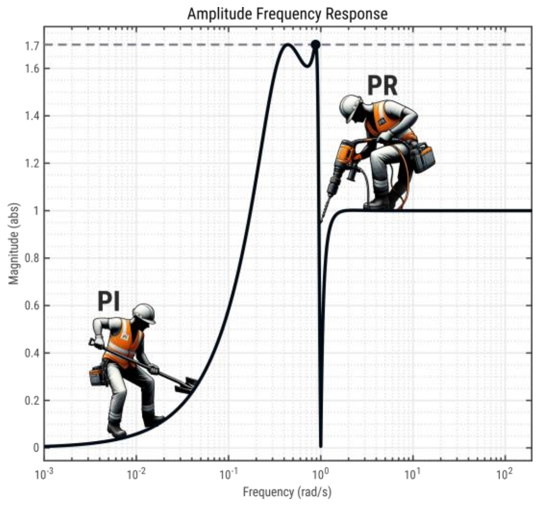
$$W(s) = \frac{s^2 + 2s + 1}{s^2 + 0.02s + 1}, \omega_0 = 1, \omega_c = 0.0004$$

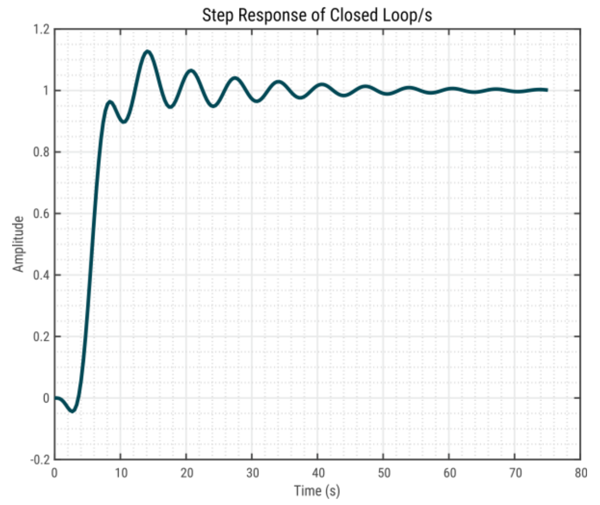
Optimal solution (maxG)

$$k_q = \text{-}0.3678,\ k_r = \text{-}446.3$$



$$C_{PI+PR}(s) = \frac{(s - 0.9028)(s + 1.0321)(s + 14.8505)}{s(s^2 + 0.0008s + 1)}$$





Example 3

This example (taken from [Å1990]) deals with the heading control of a ship moving at constant velocity. The ship's transfer function from the rudder angle to the yaw angle is $\boldsymbol{P}(s)=(b_0 s+1)b_1/s(s+a_0)(s+a_1)$. The values of its parameters depend upon the operating conditions (see Table below), including speed, trim, and loading.

Operating conditions	b_0	b_1	a_0	a_1
1	0.98	1.72	2.13	-0.325
2	1.07	1.96	1.96	-0.7
3	1.05	1.66	1.66	-0.59
4	0.93	1.86	1.86	-0.47
5	0.71	2.02	2.02	-0.21
6	0.89	2.35	2.35	0.05

Table 1: Parameter values for the ship transfer function

Find a fixed **PI controller** with static serial compensator

$$C_{Comp\text{-}PI}(s)=\frac{s^2+0.020869s+0.35551}{s^2+48.9605s+48.9181}\cdot\left(k_p+\frac{k_i}{s}\right)$$

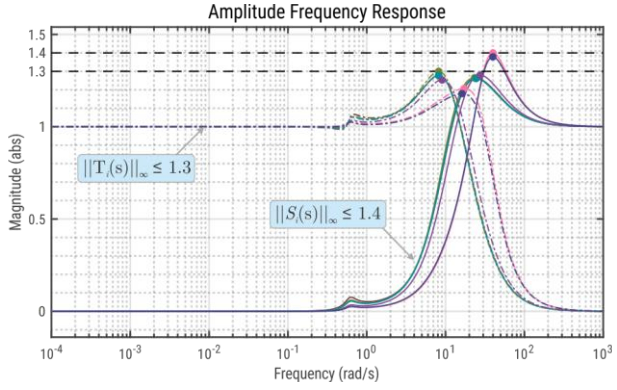
with design constraints

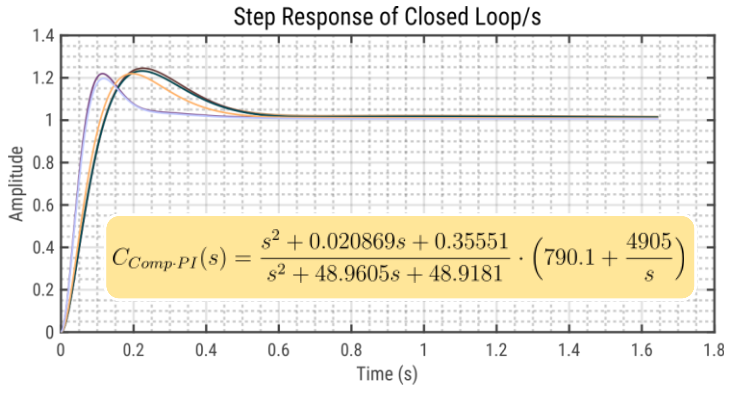
$$\|S_i(s)\|_\infty\le 1.4,\quad \|T_i(s)\|_\infty\le 1.3,\quad i=1,\dots,6$$

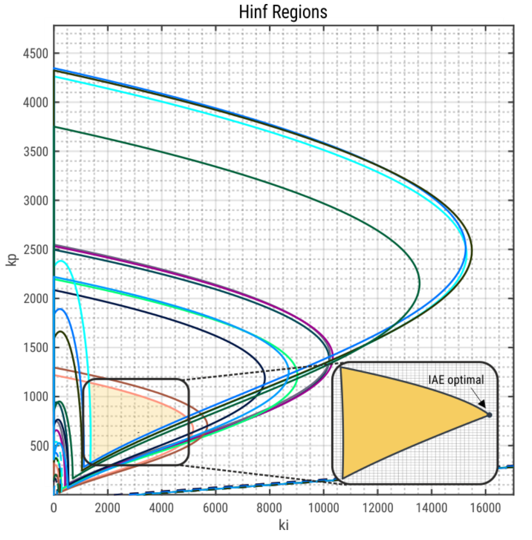
to attenuate a constant load disturbances at the plant input .

Optimal solution (IE=IAE=ITAE)

$$k_p=790.1,\ k_i=4905$$







[Å1990] Åström, K. J., Regulation of a ship's heading, 1990, In E. J. Davison (Ed.), Benchmark Problems for Control System Design. Theory Committee, IFAC.